## RESEARCH ARTICLE

# 1, 2, 3, 6: Early Gothic Architecture and Perfect Numbers 

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#### Abstract

'Knowledge of numbers should not be despised [...] We are instructed in number to avoid confusion. Take away number in all things, and they all perish. Take away computation from the world, and all things are encompassed by blind ignorance; people who are ignorant of the knowledge of reckoning cannot be distinguished from the other animals'. Thus wrote Isidore of Sevilla in the 7th century (Book III, 4). Number clearly mattered. Indeed, the medieval world seems to have taken to heart the words expressed in the Book of Wisdom (11.21): 'Omnia in mensura et numero et pondere fecisti'. In such a number-obsessed world, one would expect number to play a significant role in the design of medieval architecture, for without number, a building, like everything else, would perish. And indeed, many medieval descriptions of buildings display a great concern with numerical values. After a short introduction into the issues involved, this paper will concentrate on the use of perfect numbers as a design principle in several select examples.


## Introduction

In chapter 14 of the first part of his De arca noe morali, Hugh of St. Victor (c. 1078-1141) explains the dimensions of Noah's Ark, as given in the Bible, in a rather obscure way:

The length of three hundred cubits denotes this present age, which extends over three periods, namely, the period of natural law, the period of the written law, and the period of grace through which the Holy Church is from the world's beginning to its end advancing from this present life towards the future glory. The fifty cubits breadth denotes all believers everywhere, who are established under one Head; that is Christ. For fifty is seven times seven that is, forty-nine, the number that means the total sum of all believers - plus one, which means Christ, who is the Head of His Church and the goal of our desires. That is why the ark is gathered to one cubit at the top. The height of thirty cubits denotes the thirty volumes of the Holy Writ, namely, the twenty-two of the Old Testament and the eight of the New, wherein is contained the sum of all the things that God has either done, or else is going to do, for His Church. The three storeys signify the three ranks of believers that there are in the Church, whereof the first have commerce with the world, albeit lawfully, the second are fleeing from it and forgetting it, and the third already have forgotten it, and they are near to God. The fact that the ark gets narrower towards the top and wider below means that in the Holy Church there are more people leading a carnal life than there are

[^0]persons of a spiritual life, it being always the rule that the more perfect are proportionately few in number. The ark narrows to the measure of a single cubit at the top, because Christ, the head of his Church, who is the Saint of saints, is like to other men in all respects in nature, but in the uniqueness of His virtue He is above them all. The hundred years that the ark took to build means the same as a hundred cubits. For the hundred years signify the period of grace: since the Holy Church, which began with the beginning of the world, received redemption through the immolation of the spotless Lamb in the period of grace. (Hugh of St. Victor, translation Squire 1962: 64-65) ${ }^{1}$

This was not all Hugh has to say on the dimensions of the ark. He elaborates:
that the length of the ark is six times its width and ten times its height provides us with an allegorical figure for the human body in which Christ appeared, for it is itself His body. For the length of a body from crown to heel is six times its width from one side to the other, and its height moreover is ten times its thickness through from back to front. So if you measure a recumbent man when he is lying down quite flat, his length from head to foot is six times his width across from right to left or left to right, and ten times his thickness from the ground. Six is the number of times that fifty goes into three hundred, and there are six periods in the three ages of the world. Again, three hundred signifies faith in the Trinity, or because of the [Greek] letter Tau, of which the numerical value is 300 , and which still retains the shape of the cross among the Syrians, it signifies the cross. Fifty denotes the remission of sins, thirty the measure of the age of the fullness of

Christ. (Hugh of St. Victor, translation Squire 1962: 66-67) ${ }^{2}$

These two long passages on the measurements of the ark of Noah show that Hugh considered the numerical values provided for this structure in the Bible to have a significance elucidating the underlying symbolic meaning of the ark's structure. This type of numerological thinking was not unique to Hugh of St. Victor. ${ }^{3}$ This Parisian scholar of the 12th century joined the ranks of a distinguished series of church fathers and scholars, among whom we find Irenaeus of Lyon, St. Augustine of Hippo and St. Ambrose, Isidore of Seville, Hrabanus Maurus, Honorius of Autun, Albertus Magnus and Thomas Aquinas, all of whom had a great deal to say on number symbolism. Given the common use of this type of numerological interpretation in the Middle Ages and its application to the built structures mentioned in the Bible, it is, in my opinion, logical to presume that significant and symbolic numbers were used in contemporary church design as well, to enhance the meaning of the building and provide a sense of proportion. In this paper I set out to show that this may indeed have been the case.
First of all, I will introduce the reader to the most widely used symbolic numbers, after which I will turn to perfect numbers. After a discussion of the way numbers connected to the structure of the universe and God's plan of Creation, I will discuss a group of closely related 12thcentury churches where perfect numbers may have been used to structure the designs.

## Medieval number symbolism

To the medieval mind, numbers were imbued with symbolic qualities and could be interpreted in various ways, with a few numbers having fixed meanings (Sauer 1924; Hopper 1969). One and three stood for the undivided Trinity, and two was - amongst other interpretations connected with the two Testaments. ${ }^{4}$ The number four gave rise to more diverse interpretations, such as the four gospels, the four evangelists, the four rivers of paradise, the four directions in which the Gospel was spread, the four winds, the four elements and the four seasons (Sauer 1924: 62-66). An example of this type of interpretation is found in the writings of Irenaeus of Lyon, who remarked:

It is not possible that the Gospels can be either more or fewer in number than they are. For, since there are four zones of the world in which we live, and four principal winds, while the church is scattered throughout all the world, and the 'pillar and ground' of the church is the gospel and spirit of life; it is fitting that she should have four pillars [...] He who was manifested to men, has given us the gospels under four aspects, but bound together by one spirit [...]. For the living creatures are quadriform, and the Gospel is quadriform. (Irenaeus, ed. Roberts \& Rambaut 1868, 1: Against Heresies 3.11.8; quoted by Sunderland 1959: 96)

More ingenious is the interpretation of the number four given by Hugh of St. Victor in his Didascalicon. Hugh considered four to be the number of the soul and of the body:

> Number itself teaches us the nature of the going out and the return of the soul. Consider: Three times one makes three; three times three, nine; three times nine, twenty-seven; and three times twenty-seven, eighty-one. See how in the fourth multiplication the original 'one', or unity, recurs; and you would see the same thing happen even if you were to carry the multiplication out towards infinity; always, at every fourth stage of the process, the number 'one' recurs. Now the soul's simple essence is most appropriately expressed by 'one', which itself is also incorporeal. And the number 'three' likewise, because of the 'one' which is its constituent link, is fittingly referred to the soul, just as the number 'four', because it has two constituent links and is therefore divisible, belongs properly speaking to the body. (Hugh of St. Victor, translation Taylor 1991: 64) ${ }^{5}$

In the next chapter he continues:
For the body too they assign its number 'four'. As the figure 'one' fits the soul, so the figure 'two' fits the body. Consider: two times two makes four; two times four, eight; two times eight, sixteen; and two times sixteen, thirty-two. Here likewise in the fourth multiplication the same number whence the operation began, namely 'two' reappears, and the same thing would undoubtedly happen if one were to carry the process out to infinity; at each fourth stage of the process, the number 'two' would recur. And this is the number 'four' of the body, in which it is given to understand that everything which is composed of divisibles, or solubles, is itself also divisible, or dissoluble. (Hugh of St. Victor, translation Taylor 1991: 65-66) ${ }^{6}$

The number seven is composed of four and three, which medieval authors considered highly significant. According to St. Augustine, whose ideas had a profound impact on medieval thinking, four and three are the first even and the first odd number, ${ }^{7}$ and 'on this account it [seven] is often put for all numbers together' (St. Augustine, translation Dods 1950: 375-376). ${ }^{8}$ He adds that 'many instances are found in the holy authorities, in which the number seven is [...] commonly used to express the whole, or the completeness of anything'. Seven thus stood for nothing less than the Holy Spirit and 'perfect completeness' (St. Augustine, translation Dods 1950: 376). ${ }^{9}$ For St. Augustine seven also signified the perfection of the universal church'. Thus he writes that for this reason the apostle St. John wrote letters 'to the seven churches, thereby showing that he writes to the totality of the one Church; and in the Proverbs of Solomon it is said prefiguring this, "Wisdom hath built her house, she hath strengthened her seven
pillars"' (St. Augustine, translation Dods 1950: 575)..$^{10}$ In medieval number symbolism seven also stood for the seven gifts of the Holy Ghost, the seven virtues (divided into four cardinal and three theological virtues) and vices (Sauer 1924: 75-78; Hopper 1969: 83-85). In the Bible a great many events and mysterious circumstances are set forth by the number seven. For example, God created the seventh day on which he completed his work of creation, as a day of rest and repose, and the repose of the seventh day signifies eternal rest. The vision of John in the Apocalypse contains many references to the number seven: seven trumpets, seven blazing torches or lamps of fire, the book with the seven seals, and the lamb with seven eyes and horns (Slocum 2004: 250-251).
Eight could be the number of the Resurrection, of salvation and regeneration (Sauer 1924: 78-79; Sunderland 1959: 97). In the words of St. Augustine: 'the eighth day, which is the first day of the week, represents to us that original life, not taken away, but made eternal'. ${ }^{11}$ The number ten was also thought to be significant. Again according to St. Augustine, it denoted perfection, for it is made up of the number seven, embracing all created things, and the number three, which signified the Trinity. ${ }^{12}$ Twelve was above all the number of the Apostles, for, as St. Augustine claims, Christ chose twelve disciples 'to make known the Trinity throughout the whole world, that is, throughout the four quarters of the world. That is the reason of the three times four'. ${ }^{13}$ The apostles also stood for the twelve articles of faith, expressed in the creed, to which each of the apostles was thought to have contributed one line. ${ }^{14}$

In his Exegetica, Hugh of St. Victor analyzes nine different ways in which numbers might be meaningful: by order of position, composition, extension, disposition, computation, multiplication, according to aggregate parts, number of parts and exaggeration. ${ }^{15}$ However, as Vincent Foster Hopper remarks in his Medieval Number Symbolism of 1969, 'Hugo's attempt to organize the principles of medieval number science is itself an indication of the looseness with which these principles were applied. It was never a matter of defining the meaning of a given number by checking it against these rules, but rather of selecting the rule which would provide the traditional or desired meaning' (Hopper 1969: 103). In spite of this apparent arbitrariness in assigning significance to numbers, Hopper contends that number symbolism should be taken seriously, and did not merely constitute 'a pleasant variety of anagrams'. For the medieval scholar 'numbers were fundamental realities, alive with memories and eloquent with meaning' (Hopper 1969: VIII). As Isidore of Seville (translation Throop 2005, 1: 3.4.1) notes, 'Knowledge of numbers should not be despised. They clear up great mysteries in many parts of the Holy Scriptures'. ${ }^{16}$

## Perfect numbers

Of all the numbers with symbolic values, perfect numbers appear to have been valued more than any other numbers in the Middle Ages. A perfect number is equal to the sum of its divisors. The first three perfect numbers are 6,28
and 496. In his Etymologiae, Isidore of Seville describes the number six as follows: 'It has three parts, the sixth, the third and the half. The sixth is 1 , the third is 2 , the half is 3. These parts added make a complete 6 . The perfect numbers are: 6 within 10; 28 within a hundred; 496 within a thousand' (Isidore of Seville, translation Throop 2005, 1: 3.5.11). ${ }^{17}$

The concept of perfect numbers was first introduced in Euclid's Elements ( 300 BC ) and then taken up by Nicomachus of Gerasa (c. 100 AD ) in his Introductio arithmetica. Nicomachus held that
[o]f the simple even numbers, some are superabundant, some deficient, like extremes set over against each other, and some are intermediary between them and are called perfect. Those which are said to be opposites to one another, the superabundant and deficient, are distinguished from one another in the relation of inequality in the directions of the greater and the less[er]; for apart from these no other form of inequality could be conceived, nor could evil, disease, disproportion, unseemliness, nor any such thing, save in terms of excess and deficiency. For in the realm of the greater there arise excesses, overreaching, and superabundance, and in the less[er] need, deficiency, privation, and lack; but in that which lies between the greater and the less[er], namely, the equal, are virtues, wealth, moderation, propriety, beauty, and the like, to which the aforesaid form of number, the perfect is most akin. (Nicomachus of Gerasa, translation D'Ooge 1926: ch. 14, p. 207)

Nicomachus' Introductio was translated and slightly elaborated upon by Boethius (c. 480-524 or 525 AD) in his De institutione arithmetica, which has survived in a great number of manuscripts and must therefore have been much read. ${ }^{18}$ In it, Boethius describes the perfect and imperfect numbers in anthropomorphic terms:

So these numbers, those whose parts are added together exceed the total, are seen to be similar to someone who is born with many hands more than nature usually gives, as is the case with a giant who has a hundred hands, or three bodies joined together, such as the triple-formed Geryon. Or this number is like some monstrosity of nature which suddenly appears with a multiplicity of limbs. The other number, whose parts when totalled are less than the size of the entire number, is like one born with some limb missing, like the ugliness of the Cyclops' face. Or the number is like one who is born naturally deficient in relation to some member, who emerges short of his total fullness. Between these two kinds of number, as if between two elements unequal and intemperate, is put a number which holds the middle place between the extremes like one who seeks virtue. That number is called perfect and it does not extend in a superflu-
ous progression nor is it reduced in a contracted reduction, but it maintains the place in the middle; the sum of its parts is not more than the total nor does it suffer from a lack in comparison with the total, as are 6 and 28 [...]. There is in these a great similarity to the virtues and vices. (Boethius, translation Masi 1983: 97-98) ${ }^{19}$

In his De civitate Dei, St. Augustine (354-430) discusses the number six as the number of days of the Creation:

Six is a perfect number, not because God required a protracted time, as if He could not at once create all things, which then should mark the course of time by the movements proper to them, but because the perfection of the works was signified by the number six. For the number six is the first which is made up of its own [or aliquot] parts, i.e., of its sixth, third, and half, which are respectively one, two and three, and which make a total of six. (St. Augustine, translation Dods 1950: 374-375) ${ }^{20}$

He then goes on to explain what makes perfect numbers so special:

In this way of looking at a number, those are said to be its parts which exactly divide it, as a half, a third, a fourth, or a fraction with any denominator - e.g., four is a part of nine, but not therefore an aliquot part; but one is, for it is the ninth part; and three is, for it is the third. Yet these two parts, the ninth and the third, or one and three, are far from making its whole sum of nine. So again, in the number ten, four is a part, yet does not divide it; but one is an aliquot part, for it is a tenth; so it has a fifth, which is two; and a half which is five. But these three parts, a tenth, a fifth and a half, or one, two and five, added together, do not make ten, but eight. Of the number twelve, again, the parts added together exceed the whole; for it has a twelfth, that is, one; a sixth, or two; a fourth, which is three; a third, which is four; and a half, which is six. But one, two, three, four, and six make up, not twelve, but more, viz. sixteen. So much I have thought fit to state for the sake of illustrating the perfection of the number six, which is, as I said, the first which is made exactly up of its own parts added together; and in this number of days God finished his work. (St. Augustine, translation Dods 1950: 374-375) ${ }^{21}$

Six was thus the number of creation. No wonder God also used it and its divisors to set out the measurements of the Temple that was to be built in his honour in Jerusalem: 'And the house which King Solomon built to the Lord was sixty cubits long, twenty cubits wide, and thirty cubits high', i.e., the proportions used for the temple were 60:20:30, or, $6: 2: 3$. In his writings on the Temple of Solomon, the venerable Bede explained these numbers as follows:
[T]he length of the house is fittingly expressed by the figure of sixty cubits. For the number six whereby the world was made, conventionally denotes the perfection of good works. And it is necessary that we should endure the trials of our sojourn with forbearance so that we may deservedly be able to enter the promised land of good works when it appears. The breadth is determined by the number twenty because of the twofold dimension of the same charity wherewith we love both God and neighbour, the height is determined by the number thirty because of the belief in the Holy Trinity which is one God in the vision of whom all our hopes and desires have their fulfilment. So six has to do with the perfection of the work; two with the love of God and neighbour and three with the hope of the vision of God. Each number is rightly multiplied by ten because it is only through faith and the observance of the Decalogue of the law that our patience gets salutary exercise or our charity burns profitably or our hope is rapt aloft to yearn for the things of eternity. (Beda Venerabilis, translation Connolly \& O'Reilly 1995: 22) ${ }^{22}$

That Bede's idea was persistent can be shown with reference to Bonaventura, who in his Itinerarium mentis in Deum (1217-1274) describes the six steps (i.e., the six capabilities of the soul) leading man to enlightenment, as a microcosm echoing the six days of Creation in the macrocosm. Six here is clearly the number leading to spiritual perfection (Bonaventura, translation after Van Winden 1996: chs. 1-5, pp. 62-63).

The perfect number six even underlies the division of the Bible. According to Hugh of St. Victor, the whole of Sacred Scripture [1] is contained in two testaments [2], namely, in the Old and in the New. The books in each Testament are divided into three groups [3]. The Old Testament contains the Law, the Prophets and Hagiographers; the New contains the Gospel, the Apostles, and the Fathers [6], i.e., 1:2:3:6 (Hugh of St. Victor, translation Taylor 1991: 103). ${ }^{23}$ Seven was also thought of as a perfect number, but, as St. Augustine hastens to add, for another reason. On theological grounds, he considered seven to be a perfect number because he associated it with God's day of rest, and with completeness: 'on the seventh day [...], the rest of God is set forth, and then, too, we first hear of its being hallowed. Thus God did not wish to hallow this day by His works, but by His rest'. St. Augustine then illustrates the idea that the number seven is commonly used to express the whole, or the completeness of anything, by quoting from Scripture. He continues: 'And so the Holy Spirit, of whom the Lord says, "He will teach you all truth", is signified by this number. In it is the rest of God, the rest His people find in Him. For rest is in the whole, i.e., in perfect completeness, while in the part there is labour' (St. Augustine, translation Dods 1950: 375-376). ${ }^{24}$

Boethius notes, in what seems an apt conclusion to this section, that there are very few perfect numbers: 'You find the perfect numbers rarely, you may enumerate
them more easily, and they are produced in a very regular order. But you find superfluous or diminished numbers to be many and infinite and not disposed in any order, but arranged randomly and illogically, not generated from a certain point' (Boethius, translation Masi 1983: 98). ${ }^{25}$

## Numbers and the structure of God's universe

Numbers clearly mattered in the medieval world, which valued the words expressed in the Book of Wisdom (11:21): 'You have ordered all things in measure, and number, and weight'. ${ }^{26}$ Many medieval authors stress the importance of number as an ordering device. Exemplary is Boethius' De institutione arithmetica. The term 'arithmetic' used by Boethius did not then designate the modern discipline of counting and calculation, but denoted the theory and philosophy of number. What we call arithmetic, Boethius would have named logistics. In the Middle Ages arithmetic in our sense was known as algorism. ${ }^{27}$ Boethius claims that arithmetic (i.e., the science of numbers) is the first of the disciplines of the quadrivium to be learned, as it 'holds the principal place and position of a mother to the rest'. He also holds arithmetic to be
prior to all not only because God, the creator of the massive structure of the world, considered this first discipline as the exemplar of his own thought and established all things in accord with it; or that through numbers of an assigned order all things exhibiting the logic of their maker found concord; but arithmetic is said to be the first for this reason also, because whatever things are prior in nature, it is to these underlying elements that the posterior elements can be referred. (Boethius, translation Masi 1983: 74) ${ }^{28}$

He concludes:

From the beginning, all things that have been created may be seen by the nature of things to be formed by reason of numbers. Number was the principal exemplar in the mind of the creator. From it was derived the multiplicity of the four elements, from it were derived the changes of the seasons, from it the movement of the stars and the turning of the heavens. (Boethius, translation Masi 1983: 75-76) ${ }^{29}$

In the seventh century, Isidore of Seville claimed that 'we are instructed in number to avoid confusion. Take away number in all things, and they all perish. Take away computation from the world, and all things are encompassed by blind ignorance; people who are ignorant of the knowledge of reckoning cannot be distinguished from the other animals' (Isidore of Seville, translation Throop 2005: 3.4.4). ${ }^{30}$ In the 10th century, Hroswitha of Gandersheim wrote, 'The more a man realizes the wonderful way in which God has set all things in number and measure and weight, the more ardent his love (Hroswitha of Gandersheim, ed. St. John 1923: act 1, p. 101). ${ }^{31}$ That
such ideas with regard to numbers persisted for centuries can be shown with reference to Bonaventura's 13th-century Itinerarium mentis in Deum, which states:
[Since] there is no beauty and pleasure without proportion, and proportion is to be found primarily in numbers; all things must have numerical proportion. Consequently, number is the principal exemplar in the mind of the Creator and as such it is the principal vehicle that, in things, leads to wisdom. Since this device is extremely clear to all and is closest to God, it leads us to Him through seven stages and it causes us to know Him in all corporeal and sensible things; and while we learn that things have numerical proportion, we take pleasure in this numerical proportion and we judge things irrefutably by virtue of the laws that govern it. (Bonaventura, translation after Van Winden 1996: 90-91) ${ }^{32}$

In a world so preoccupied with number, its significance and its structuring principle, it would seem logical for number - and perfect and symbolic numbers in particular - to have played a significant role in the arts of the time and in the design of medieval architecture. Could it be that number was intentionally used as a designing principle or played a role in design? Indicative of the use of number in medieval art is the splendid 'In Principio' page at the beginning of the Book of Genesis in the Bible of St. Hubert ${ }^{33}$ of circa 1085, in the Royal Library of Belgium, to which Harry Bober drew attention in 1961 (Fig. 1).

In this Mosan miniature, the figure of Christ is shown in a roundel placed at the intersection of the overlapping initials IN. Surrounding this bust are four more roundels with figures personifying the four elements. Each element is identified by an inscription. Along the frame of each medallion, a Roman numeral attributes a number to each element. Below and on the sides the divisors of this numeral are written out in capital letters. The numbers given are: $2 \times 2 \times 2=8$ (fire), $2 \times 2 \times 3=12$ (air), $2 \times 3$ x $3=18$ (water) and $3 \times 3 \times 3=27$ (earth). Bober terms this number theory 'arithmetic theology' or 'arithmology', a discipline concerned with the 'properties, virtues, and powers of numbers and geometric forms for their bearing on the origins and nature of the universe' (Bober 1961, 1: 13-28). ${ }^{34}$

## The use of symbolic and perfect numbers in architecture

There are several buildings in which architectural historians have noted that number symbolism played an important role in the design. ${ }^{35}$ A well-known example of such a building is Abbot Suger's Saint-Denis near Paris. In his De consecratione, the abbot mentions the twelve columns of the nave of his newly-rebuilt church as representing 'the number of the Twelve Apostles' and the columns in the side aisles 'the number of the [minor] prophets, according to the Apostle who builds spiritually' (Suger of St. Denis, translation Panofsky 1979: 104-105). ${ }^{36}$ This symbolic interpretation


Fig. 1: 'In Principio' page at the beginning of the Book of Genesis in the Bible of St. Hubert of circa 1085. Brussels, Royal Library of Belgium, ms. II. 1639, fol. 6v.
was not unique to Abbot Suger and should be seen in the context of the two meanings of the word 'ecclesia'. A church was, of course, the material building in which the divine office was read, but it was also a spiritual building. This double interpretation is very clearly expressed in William Durandus' (1230-1296) treatise Rationale Divinorum Officiorum (Book I-1), in which it is noted:

The word church has two meanings: the one, a material building, wherein the divine offices are celebrated; the other, a divine spiritual fabric, which is the collection of the faithful. The Church, that is the people forming it, is assembled by its ministers, and collected together into one place by 'Him who makes men to be of one mind in one house'. For as the material church is constructed from the joining together of various stones, so is the spiritual Church by that of various men. (Durandus, translation Fons Vitae 2007: 3)

Like Suger, Durandus furthermore holds that

The piers of the church are the bishops and doctors who especially sustain the Church of God by their doctrine [...]. The bases of the church are the apostolic bishops who support the frame of the whole
church. The capitals of the piers are the opinions of the bishops and doctors. For as the members are directed and moved by the head, so are our words and works governed by the mind. (Durandus, translation Fons Vitae 2007: 18-19)

In the church of St. Michael in Hildesheim, built by Bishop Bernward (ca. 960-1022) from circa 1000 onwards, number symbolism also played an important role in the design. This church has a nave of three bays, with four piers, and twelve columns in total, thus representing the Trinity, four evangelists and twelve apostles. To give substance to the symbolic meaning of the church as a living 'Ecclesia' founded on the apostles and saints, relics of saints were placed inside the impost blocks above the capitals Schuffels (1993: 538-540); Bosman (1985: 31-32). ${ }^{37}$ In 1956, Otto von Simson (1974: 212-214) suggested that number also played an important role in the design of the north and south transept arms of St. Michael's, each of which has two galleries, superimposed upon the double arcade on the ground level. Each bay of the ground level thus has one arch, with two above it and then three above them. Although Von Simson interpreted these ratios as musical, in my view it is more significant that one, two and three are the divisors of the perfect number six (Fig. 2).

By applying the number of creation to each transept arm, the symbolic meaning of the building as the house of God was enhanced. The perfection of the building and what it stood for was underscored by the use of the perfect number six and its divisors.

In 1990, Christopher Wilson (1990: 64-66) drew attention to the apparently conscious application of numerical values in the choir of St. Rémi in Reims, built under the abbacy of Pierre de Celle (from circa 1162 until 1181, when he became bishop of Chartres) and his successor, Simon (1181-1198) Prache (1978: 42). ${ }^{38}$ This choir was the first to have triple lancets in the clerestory. 'Like most churchmen of his day', Wilson writes, 'Pierre de Celle, would have been familiar with the passages in St. Augustine's writing which deal with the symbolism of numbers. For Augustine, six was the perfect number because the works of Creation were completed (perfecta) in six days. Six also expresses the perfection of the Creation itself since it is the first number comprising the sum of its divisors, that is its fractions: one, two and three - a sixth, a third and a half. The succession of arches in the arcade, gallery and clerestory of each bay at St. Rémi embodies this equation exactly, with the most elevated number allotted to the highest position' (Fig. 3) (Wilson 1990: 64-66). ${ }^{39}$

Wilson also states that it is probably not coincidental that the number three is placed the highest, for this number stands for the Trinity. According to Wilson, such Trinitarian symbolism was quite common at the time, and he refers to the 12th-century scholar Rupert of Deutz, who held that the tripartite elevation referred to the Trinity, and to Abbot Suger of Saint-Denis who completed the chevet of St. Denis in precisely three years and three months. ${ }^{40}$ In addition, Wilson mentions that there are 11 bays in the St. Rémi choir, adding up to a total


Fig. 2: The transept elevation of the church of St. Michael in Hildesheim. Photo: author.
of 33 windows in the choir clerestory, a number that referred to Christ, who spent 33 years on earth (Figs. 3 and 4; Wilson 1990: 66).

There are thus two systems at work here. The number of arches in each bay ( $1+2+3$, from bottom to top) is based upon the perfect number 6 and its divisors, thus denoting perfection; at the same time the numbers 3 and 33 have symbolic meanings derived from scripture.

In addition to what Wilson wrote about the glazing programme at St. Rémi, it should be noted that the 33 windows of the clerestory, which were unfortunately lost during 19th-century alterations and bombing in the Second World War, showed on its axis the figure of the enthroned Virgin with child ( 1 window). She was flanked by the apostles ( 12 windows), evangelists ( 4 windows) and precursors of Christ ( 16 windows). Below these biblical figures were the saints and bishops of the diocese of Reims, with St. Rémi taking the position under the Virgin Mary. The central window of the tribune is still there and shows the Crucifixion (Prache 1978: 73; Grodecki 1983, 2: 130-140).

The idea that number symbolism was used in the choir elevation of St. Rémi is compelling and raises the question of whether there are other examples of early-Gothic


Fig. 3: The choir of the abbey church of St. Rémi in Reims, built under the abbacy of Pierre de Celle (from circa 1162 until 1181, when he became bishop of Chartres) and his successor Simon (1181-1198). Photo: author.
architecture where use might have been made of numerical systems based on perfect numbers such as 6 and 28 and their divisors, and on the number 7. In the following paragraphs I would like to introduce three more examples of parts of buildings where this may have been the case.

## Notre-Dame-en-Vaux at Châlons-en-Champagne

The choir of the parish and collegiate church of Notre-Dame-en-Vaux in Châlons-en-Champagne (formerly Châlons-sur-Marne) was rebuilt, probably from circa 1187, and was ready for use in 1217 (Figs. 5, 6). This choir is very similar to that of Saint-Rémi in Reims and has been regarded as a reduced version of it by Demaison (1899: 84-107), Prache (1966: 1-64), and Corsepius (1997: 121 and 161-167). This apparent reduction they attributed to the restricted space available for building and the existence of prior structures, which also caused the elevation of the choir bays to be irregular. The central part of the Châlons choir consists of a single choir bay and a fivesided termination, and is thus less deep than that of St. Rémi. The central bay shows the same system as St. Rémi: on the ground level is one arch (1), on the tribune level are two arches (2), the triforium level has six arches (or, 2 x 3 arches $=6$ ) and the clerestory has three arches (3), but


Fig. 4: The choir clerestory of the abbey church of St. Rémi in Reims. Photo: author.
the flanking two bays have two rather than three windows in the clerestory. The next bays again have three windows in the clerestory (Fig. 6). This Châlons arrangement thus gives very different numerical values than the St. Rémi elevation. The perfect number 6 is present in the vertical axis only of the central bay (Fig. 5). The three windows at the top likely refer to the Trinity. The two flanking clerestory bays have a total of four windows ( $2 \times 2=4$ arches), a number that perhaps represents the four evangelists. The next two pairs of clerestory bays contain six windows per side, making a total of twelve, the number of the Apostles. ${ }^{41}$ The total of nineteen windows has no apparent significance. Here then, a combination of perfect number for the vertical axis and a series of symbolic numbers for the horizontal arrangement of the upper windows could have been used.

## Noyon Cathedral

Dieter Kimpel and Robert Suckale (1985) note that the elevation of the southern transept of Noyon Cathedral, completed circa $1185,{ }^{42}$ was unusual when compared to other contemporary buildings because, being a processional space through which the bishop entered the cathedral (the canons used the north transept), it was built with no tribune. Due to the absence of the tribune the architect


Fig. 5: The choir of the parish and collegiate church of Notre-Dame-en-Vaux in Châlons-en-Champagne, which was rebuilt, probably from circa 1187 onwards and which was ready for use in 1217. Photo: author.
was able to situate the triforium at an unusually low level. Kimpel and Suckale know of no functional reasons for this deviation and therefore assume that the motivation was purely aesthetic. Art historians, they write, have always taken offence to this juggling around of architectural elements and have seen in this nothing but experimenting, which they (these art historians) considered to be an indication that in this part of Noyon Cathedral early Gothic had not as yet reached perfection. For this reason, still according to Kimpel and Suckale, the Noyon south transept is little known and seldom reproduced in the art and architectural history literature. However, they write, when one leaves behind this art-historical notion of a 'classical' look ('Klassizität') of the three storeyed Gothic elevation and when one takes the building for what it is, it is at once apparent that it is one of the most brilliant (glänzendsten) architectural compositions of the Gothic era. ${ }^{43}$ What they do not explain is why this might be so. By what means did the architect achieve this harmonious composition?
The elevation of the Noyon south transept differs from that of every other early Gothic elevation. It is also unlike that of the choir and north transept of Noyon Cathedral. As mentioned, the bishop entered the church via the south transept. Interestingly, the choir has 28 windows in its double clerestory (Figs. 7, 8). Twenty-eight is the


Fig. 6: The choir clerestory of Notre-Dame-en-Vaux in Châlons-en-Champagne. Photo: author.


Fig. 7: The four-storeyed south transept of the cathedral of Noyon. Photo: author.
perfect number of 100, as noted above. Twenty-eight has the divisors $1,2,4,7$ and 14 and all these numbers can be traced in the design of the transept. The four-storey elevation of the transept has seven bays, and within each bay is one window (1) with two arches below on the ground floor level (2), a triforium with a triplet of arches (3), and a lower and upper clerestory with two windows each $(2+2=4)$. The sum of the openings of the triforium and clerestory per bay is 7 . In all, each bay has a total of 10 arches and windows (i.e., from the bottom: $2+1+3+2$ $+2=10)$. The bays are unified across the transept by two ranges of 14 windows, for a total of 28 . Thus the perfect number 28 and its divisors could very well underlie the number of arches and windows in the apse of the south transept of Noyon Cathedral to imbue the building with significance.

## Canterbury Cathedral, the Corona

The perfect number 28 may also underlie the design of Thomas Becket's corona at Canterbury Cathedral, which houses the top part of the saint's cranium that was struck off during his martyrdom on 29 December 1170. Twentyeight, the perfect number of 100 , is made up of 4 times 7 . The present corona is a seven-sided rotunda, not counting the extra wide entrance side (Figs. 9, 10). It has a tripartite elevation, with each bay having a pointed window at the lower level, a double-arched triforium in the middle, and a pointed clerestory window on top. There are thus a total of seven arches in the lower level, fourteen in the middle and seven on top, adding up to $7+14$ $+7=28$ arches. Although the number of arches fits this scheme perfectly, there are only five windows in both the


Fig. 8: The south transept clerestory of Noyon cathedral. Photo: author.


Fig. 9: The corona of Canterbury Cathedral. Photo: author.
ground floor and clerestory, for a total of ten windows; because of the flanking towers, the arches on the ends are blind.

The number seven would have befitted all saints, for it denoted the seven gifts of the Holy Spirit as well as the seven virtues, but in the case of Thomas Becket, canonized in 1173 , there is more. As Kay Brainerd Slocum shows in her book, Liturgies in Honour of Thomas Becket, the Canterbury monks were very interested in auspicious numbers and laid great stress on Becket's relation with the number seven. Brainerd Slocum notes that the saint's relics were translated to the choir on 7 July 1220 in the presence of King Henry III and many lay and ecclesiastical worthies. The translation date, 7 July, was designated a feast day to be observed each year in many areas of England and Europe (Slocum 2004: 242). Most importantly, in her view, the original five auspicious Tuesdays in Becket's life were augmented to seven, by the addition of the visit of Thomas to Pontigny (of which the actual date is unknown) and by placing the translation day, the seventh auspicious Tuesday, on the seventh day of July (Slocum 2004: 249). The seven portentous Tuesdays of Becket's life are mentioned in Lesson 3 of the Office for the Translation, as found in the British Library Stowe 12 and Cotton Appendix 23 and in Lessons 5 and 6 of the Sarum Breviary. ${ }^{44}$ Lesson 3 of the Office of the Translation ends with the following passage: ‘These seven deeds [that occurred] on Tuesdays were not without premonition; on the basis of these seven events one can understand what the Lord says through his prophet, "Above one stone there


Fig. 10: The clerestory of the corona of Canterbury cathedral. Photo: author.
are seven eyes"'. ${ }^{45}$ The same biblical passage is used in connection with the seven Tuesdays in Lessons 5 and 6 of the Sarum Breviary. ${ }^{46}$ Brainerd Slocum describes the meaning of the number seven as follows:

According to exegetical tradition, the seven eyes represent the manifold providence of Christ over his Church, or the seven gifts of the spirit of God. The number seven is also used in scripture as a number of completion and perfection; as the 'perfect' number, it implies a fullness or completeness of spiritual endowments. [...] Thomas' life was marked by the seven Tuesdays, and on the seventh, the Translation, he was 'completed' so to speak, and arrived at his place of eternal rest. (Slocum 2004: 250-251)

The Canterbury monks clearly considered the number seven to be important and befitting their most famous martyr. It would be entirely fitting, therefore, if it also played an important role in the design of the corona.

## Conclusion

As Durandus noted in the 13th century,
everything that pertains to divine worship, the practices and vestments used by the Church, are full of divine meanings and mysteries. Each and every one of them, when examined with care and love by an individual who knows how to draw
honey from stones and oil from the hardest of rocks, pours forth a celestial fragrance. (Durandus, translation Fons Vitae 2007: ch. 19, proeme 1) ${ }^{47}$

The musings of Hugh of St. Victor over the significance of the Biblical measures of Noah's Ark are clearly those of an individual who could extract honey from stones and his interpretation of the ark's measurements would not have been common knowledge, and so it is with many other forms of number symbolism. Apart from interpretations of the number three for the Trinity, four for the four evangelists and twelve for the apostles, knowledge of number symbolism and perfect numbers would have been restricted to the intellectual elite. Interestingly, with the choirs of St. Rémi at Reims and Notre-Dame-en-Vaux at Châlons-en-Champagne, the south transept of Noyon Cathedral and the Canterbury Corona we are in fact dealing with spaces intended for a restricted, mainly ecclesiastical audience. If number symbolism were indeed applied here such an audience could have comprehended and interpreted the intended symbolism. The writings of the same Durandus, however, which I have quoted above, should be seen as a caution against reading too much into a church structure. In his extensive treatise Rationale Divinorum Officiorum, everything that has to do with the church building, its furnishings and the divine office is explained in a symbolic manner. Most of these explanations do not strike the reader as having been intended from the start, but rather have the character of explanations that were conjured up 'après la pose'. In the absence
of documents dealing with the designs of these early Gothic elevations, it is impossible to prove that numerology was consciously and deliberately used as a design feature. Given the strong medieval interest in number symbolism, as revealed in particular by medieval interpretations of the biblical dimensions, in terms of numbers of cubits, of structures such as Noah's Ark and Solomon's Temple or the use of the number seven in relation to the cult of Thomas Becket, intended number symbolism in early Gothic church architecture would seem to be more likely than not.

## Notes

${ }^{1}$ Hugh of St. Victor (1854, PL 176: 630) 'Trecentorum cubitorum longitude, praesens saeculum designat, quod tribus temporis decurrit, id est tempore naturalis legis, tempore scriptae legis, tempore gratiae, per quae sancta Ecclesia a principio mundi usque ad finem, a praesenti vita ad futuram gloriam tendit. Quinquaginta cubitorum latitude universos fideles significant, qui sub uno capite sunt constitute, id est Christo. Nam quinquaginta constat ex septies septem, id est quadraginta novem, qui numerus universitatem fidelium designat, et uno superaddito, quod significat Christum qui est caput Ecclesiae suae (Ephes. V), et finis desideriorum nostrorum. Propter quod in uno cubito consummatur. Altitudo tringinta cubitorum significat tringinta volumina divinae paginae, id est viginti duo Veteris Testamenti, et octo Novi, in quibus summs omnium continetur, quae deus vel fecit vel facturus est propter Ecclesiam suam. Tres maniosnes significant tres ordines fidelium, qui sunt in sancta Ecclesia, quorum primi utuntur mundo, licite tamen. Secundi fugiunt et obliviscuntur mundum. Tertii jam obliti sunt mundum, et ii sunt propinqui Deo. Quod in superioribus arca contrahitur, in inferioribus dilatatur, hoc significat, quod in sancta ecclesia major est numerus carnalem vitam ducentium quam spiritualium, ita ut semper quanto perfectiores, tanto sint pauciores. In supremo ad unius cubiti mensura marca contrahitur, quia Christus caput Ecclesiae suae, qui est Sanctus sanctorum, per naturam consimilem inter homines est et per virtutem singularem supra homines est. Centum anni, in quibus aedificata est arca, idem significant quod centum cubiti. Nam centum anni significant tempus gratiae, quia sancta Ecclesia, quae ab initio mundi coepit, in tempore gratiae per immolationem Agni immaculati redemptionem accepit'. (De arca noe morali 1.4)
${ }^{2}$ Hugh of St. Victor (1854, PL 176: 631): ‘Quod vero sexies longa est ad latitudinem suam haec arca, et decies ad altitudinem, suam, humani corporis instar ostendit, in quo Christus apparuit; nam et ipsa corpus ejus est. Corporis enim longitudino a vertice usque ad vestigium sexies tantum habet quantum latitude, quae est ab uno latere usque ad alterum latus, et decies tantum quantum et altitude, cujus altitudinis mensura est in latere a dorso in ventrem, velut si jacentem hominem metiaris supinum sive pronum, sexies tantum longitudinis est a capite usque ad pedes, quantum latus a
dextra ad sinistram, vel a sinistra in dexteram, et decies quantum altus a terra. Sexies est numeros quinquagenarius in trecentis, et sex aetates sunt in tribus temporibus saeculi. Vel trecenti significant fidem Trinitatis, vel crucem proper Tau, quod in numeris trecentos significat, et adhuc apud suos formam retinet crucis. Quinquaginta remissionem peccatorum designant, triginta mensuram aetatis plenitudinis Christi.' (De arca noe morali 1.4)
${ }^{3}$ Hugh's explication of the ark's dimensions is to a large extent taken from St. Augustine.
${ }^{4}$ One: Sauer (1924: 69), who refers to St. Augustine (De libero arbitrio 2.16, and Dialog. Quaest. 26) and Durandus (Rationale 3.14). Three: Sauer (1924: 70-72). Two: Sauer (1924: 69-70), who refers to St. Augustine (Contra Faust. 22.52) and Pope Innocentius III (De sacro altari mysterio 1.59).
${ }^{5}$ Hugh of St. Victor (1939: II-4): 'Huius quoque progressionis regressionisque rationem ipse etiam numerus docet. dic: 'ter unum fiunt tria", dic: '"er tria fiunt novem", dic: "ter novem fiunt viginti septem", dic: "ter viginti septem fiunt octoginta unum". Ecce tibi in quarto gradu unitas prima occurrit, idemque evenire videbis, si usque ad infinitum duxeris multiplicationem, ut semper in quarto gradu unitas emineat. Rectissime autem simplex animae essentia unitate exprimitur, quae ipsa quoque incorporea est. ternarius quoque propter indissolubile mediae unitatis vinculum congrue ad animam refertur, sicut quaternarius, quia duo media habet ideoque dissolubilis est, proprie ad corpus pertinent'.
${ }^{6}$ Hugh of St. Victor (1939: II-5): 'Nam corpori quoque suum assignant quaternarium. sicut monas animae, ita dias corpori congruit. dic: "bis duo fiunt quattuor", dic: "bis quattuor fiunt octo", dic: "bis octo fiunt sedecim", dic: "bis sedecim fiunt triginta duo". Hic in quarto loco similiter idem numerus, id est binarius, a quo multiplicatio initium sumpsit, tibi occurrit, idemque si in infinitum processeris, indubitanter continget ut quarto semper gradu binarius emineat. et hic est quaternarius corporis, in quo intelligi datur omne quod a solubilibus compositionem accipit ipsum quoque esse dissolubile.'
${ }^{7}$ Augustine does not consider one and two to be real numbers. As Elizabeth Sunderland explains (1959: 96 note 12), 'three and four were major numbers in the Pythagorean system because the Pythagoreans conceived of mathematics in geometric terms. Thus one was thought of as a point, two as two points, etc. Two points joined make a line, but with three points can be made a triangle and with four points a pyramid. Because three is the smallest number which can represent surface it was thought of as the first 'real' number, and four the second real number because it can represent a solid'.
${ }^{8}$ St. Augustine (1845: 344-345): 'Hoc itaque satis sit admonere, quod totus impar primus numerous ternarius est, totus par quaternarius: ex quibus duobus septenarius constat. Ideo pro universo saepe ponitur, sicuti est [...].' (De civitate Dei 11.31)
${ }^{9}$ St. Augustine (1845: 345): 'Et multa hujusmodi in divinis auctoritatibus reperiuntur, in quibus septenarius numerous, ut dici, pro cujusque rei universitate poni solet. Propter hoc eodem saepe numero significatur Spiritus sanctus [...]. In toto quippe, id est in plena perfectione [...].' (De civitate Dei 11.31).
${ }^{10}$ St. Augustine (1845: 529): 'Hic totum quod prophetabatur eluxit agnoscentibus numerum septenarium, quo est universae Ecclesiae significata perfectio. Propter quod est Joannes apostolus ad septem scribit Ecclesias (Apoc. I: 4), eo modo se ostendens ad unius plenitudinem scribere: et in Proverbiis Salomonis hoc autea praefigurans Sapientia, Aedificavit sibi domum, et suffulsit columnae septem (Prov. IX:1).' (De civitate Dei, 17.4)
${ }^{11}$ Works of St. Augustine, 'Letter LV’, chapter IX-16 and chapter X-17. Quoted by Sunderland (1959: 97).
${ }^{12}$ Works of St. Augustine, 'Reply to Manichaeus' Fundamental Epistle' X-11. Quoted by Sunderland (1959: 97). See also Sauer (1924: 80-81).
${ }^{13}$ Works of St. Augustine, 'On the Gospel of John', tract. XLIX-8. Quoted by Sunderland (1959: 97).
${ }^{14}$ See Sauer (1924: 66-69) for other, less common, medieval interpretations of the number twelve.
${ }^{15}$ Hugh of St. Victor (1854, PL 175: 22-23; In scripturam sacram 15): 'Numeri igitur novem modis significant in divino eloquio: secumdum ordinem positionis, secundum qualitatem compositionis, secundum modum porrectionis, secumdum formam dispositionis, secundum computationem, secumdum multiplicationem, secundum partium aggregationem, secundum multitudinem, secundum exaggeraionem'; Hopper (1969: 100-103).
${ }^{16}$ Isidore of Seville (1911: 3.4.1): 'Ratio numerorum comtemnenda non est. In multis enim sanctarum scriptuarum locis quantum mysterium habent elucet'.
${ }^{17}$ Isidore of Seville (1911: 3.5.11): 'habet enim tres partes, sextam, tertiam, [et] dimidiam: sexta eius unum est, tertia duo, dimidia 15 tres. Haec partes in summam ductae, id est unum et duo et tria simul eundem consummant perficiuntque senarium. Sunt autem perfecti numeri intra denarium vi, intra centenarium xxviii, intra millenarium ccccxcvi'.
${ }^{18}$ For the number of manuscripts containing Boethius' De institutione arithmetica, see Boethius (1983: 58-63).
${ }^{19}$ Boethius (1867: 40-41; De institutione arithmatica 1.19-20): 'Atque hi quidem hoc modo sunt, ut prior ille, quem suae partes superant, talis videatur, tamquam si quis multis super naturam minibus natus, ut centimanus gigas vel triplici coniunctus corpore, ut Geryo tergeminus, vel quicquid unquam monstruosum naturae in partium multiplication subripuit; ille vero, ut si naturaliter quadam necessaria parte detracta aut minus oculo nasceretur, ut Cyclopeae frontis dedecus fuit, vel quo alio curtatus membro natural totius suae plenitudinis dispendium sortiretur. Inter hos, autem velut inter inaequales intemperantias medii temperamentum limitis sortitus est ille numerous, qui perfectus dicitur, virtutis scilicet aemulator, qui nec supervacua progression porrigitur, nec contracta rursus deminutione remittitur, sed medietatis
obtinens terminum suis aequus partibus nec crassatur abundantia, nec eget inopia, ut VI vel XXVIII [...]. Est autem in his quoque magna similitude virtutis et vitii'.
${ }^{20}$ St. Augustine (1845: 343-344; De civitate Dei 11.30): 'haec autem propter senarii numeri perfectionem, eodem die sexies repetition, sex diebus perfecta narrantur: non quia Deo necessaria fuerit mora temporum, quasi qui non potuerit creare Omnia simul, quae deinceps congruis motibus peragerent tempora; des quia per senarium numerum est operum significata perfectio. Numerus quipped senarius primus completur suis partibus, id est, sexta sui parte, et tertia, et dimidia, quae sunt unum, et duo, et tria : quae in summam ducta, sex fiunt'.
${ }^{21}$ St. Augustine (1845: 344; De civitate Dei 11.30): 'Partes autem in hac consideratione numerorum illae intelligendae sunt, quae quotae sint dici potest: sicut dimidia, tertia, quarta, et deinceps ab aliquo numero denominate. Neque enim, exempli gratia, quia in novernario numero quatuor pars aliqua ejus est, ideo dici potest quota ejus sit: unum autem potest, nam nona ejus est; et tria potest, nam tertia ejus est. Conjunctae vero istae duae partus ejus, nona scilicet atque tertia, id est, unum et tria, lone sunt a tota summa ejus, quod est novem. Itemque in denario quaternarius est aliqua pars ejus; sed quota sit dici non potest: unum autem potest; nam decima pars ejus est. Habet et quintam, quod sunt duo: habet et dimidiam, quod sunt quinque. Sed hae tres partes ejus, decima et quinta et demidia, id est unum et duo et quinque, simul ductae non complent decem : sunt enim octo. Duodenarii vero partes numeri in summam ductae, transeunt eum: habet enim duodecimam, quod est umum; habet sextam, quae sunt duo; habet quartam, quae sunt tria; habet tertiam quae sunt quatuor; habet et dimidiam, quae sunt sex, non duodecim, sed amplius, id est, sexdeeim fiunt. Hoc breviter commemorandum putavi ad commendandam senarii numeri perfectionem, qui primus, ut dici, partibus suis in summam redactis ipse perficitur: in quo perfecit Deus opera sua'.
${ }^{22}$ Beda Venerabilis (1862: 749, De Templo Salomonis Liber 6-1): ‘Unde apte longitude domus, sexagenario cubitorum numero comprehenditur. Senarius namque numerous, in quo mundus factus est, perfectionem solet operum designare bonorum. Et necesse est nos ita per longanimitatem molestias ferre nostrae peregrinationis, ut merito bonae operationis promissam patriam, cum apperuit, valeamus interare. Latitudo vicenario numero determinatur, propter geminam ejusdem charitatis, qua Deum et proximum amamus, distantiam. Altitudo tricenario, propter fidem sanctae Trinitatis, qui [quae] unus est Deus, in cujus visionem cuncta spei nostrae desideria suspenduntur. Sex ergo ad operis perfectionem, duo ad dilectionem Dei et proximi, tria pertinent ad spem divinae visionis. Singuli autem numeri recte per decem multiplicantur, quia non nisi per fidem et custodiam decalegi legis, vel patientia nostra salubriter exercetur, vel charitas utiliter ardescit, vel spes sublimiter ad aeterna desideria rapitur'.
${ }^{23}$ Hugh of St. Victor (1939: 4.2): ‘Omnis divina scriptura in duobus testamentis continetur, in veteri videlicet et novo. utrumque testamentum tribus ordinibus distinguitur. Vetus Testamentum continet legem, prophetas, hagiographos, Novum autem evangelium, apostolos, patres.'
${ }^{24}$ St. Augustine (1845: 344; De civitate Dei 11.31) 'In septimo autem die, id est eodem septies repetito, qui numeros etiam ipse alia ratione perfectus est, Dei requies commendatur, in qua primum sanctificatio sonat. Ita Deus noluit istum diem in ullis suis operibus sanctificare, sed in requie sua, quae non habet vesperam [...] ' and 'Propter hoc eodem saepe numero significatur Spiritus sanctus, de quo Dominus alt, Docebit vos omnem veritatem (Joan. XVI:13). Ibi requies Dei, qua requiescitur in Deo. In toto quippe, id est in plena perfection, requies; in parte autem labor'.
${ }^{25}$ Boethius (1867: 41-42; De institutione arithmatica 1.20): 'perfectos enim numerous rarenter invenies, eosque facile numerabiles, quipped qui pauci sint et nimis constant ordine procreati. At vero superfluous ac deminutos longe multos infinitosque repperies, nec ullis ordinibus passim inordinateque dispositos et a nullo certo fine generatos'.
${ }^{26}$ Book of Wisdom (11:21): 'Omnia in mensura et numero et pondere fecisti'.
${ }^{27}$ Masi in Boethius (1983: 11): 'The modern meaning of arithmetic conveys nothing of what it meant for Boethius. The difference between arithmetic ( $\alpha \varrho \iota \theta \mu \eta \tau \iota \kappa \eta ́)$ and logistics ( $\lambda$ о $\gamma \iota \sigma \tau \iota \kappa \eta$ ) was the same for Boethius as it was for the Greeks who originally defined it. Both disciplines deal with numbers, but arithmetic designates the theory or philosophy of number; only after the Middle Ages did the term designate an elementary discipline of counting and calculation. The process whereby one undertook the solution of practical problems of computation was known to the Greeks and to Boethius as logistics and to the Middle Ages as algorism'.
${ }^{28}$ Boethius (1867: 10; De institutione arithmatica I-1): 'Haec enim cunctis prior est, non modo quod hanc ille huius mundanae molis conditor deus primam suae habuit ratiocinationis exemplar et ad hanc cuncta constituit, quaecunque fabricante ratione per numeros adsignati ordinis invenere concordiam, sed hoc quoque prior arithmetica declaratur, quod, quaecunque natura priora sunt, his sublatis simul posteriora tolluntur'.
${ }^{29}$ Boethius (1867: 12; De institutione arithmatica I-1): 'Omnia quacunque a primaeva rerum natura constructa sunt, numerorum videntur ratione fomata. Hoc enim fuit principale in animo conditoris exemplar. Hinc enim quattuor elementorum multitudo mutuata est, hinc temporum vices, hinc motus astrorum caelique conversio'.
${ }^{30}$ Isidore of Seville (1911: 3.4.4) ‘Tolle numerum in rebus omnibus, et omnia pereunt. Adime saeculo conputum, et cuncta ignorantia caeca conplectitur, nec differri potest a ceteris animalibus, qui calculi nesciunt rationem'.
${ }^{31}$ Hroswitha of Gandersheim (1853: 1032): 'Quanto enim mirabiliori lege Deum Omnia in numero et mensura et pondere posuisse quis agnoscit, tanto in ejus amore ardescit'.
${ }^{32}$ Bonaventura (1996: 90-91; Itinerarium mentis in Deum, II-10): ‘Cum igitur omnia sint pulcra et quodam modo delectabilia; et pulcritudo et delectatio non sint absque proportione; et proportio primo sit in numeris: necesse est, omnia esse numerosa; ac per hoc "numerus est praecipuum in animo Conditoris exemplar" et in rebus praecipuum vestigium ducens in Sapientiam. Quod cum sit omnibus evidentissimum et Deo propinquissimum, propinquissime quasi per septem differentias ducit in Deum et facit, eum cognosci in cunctis corporalibus et sensibilibus, dum numerosa apprehendimus, in numerosis proportionibus delectamur et per numerosarum proportionum leges irrefragabiliter iudicamus'.
${ }^{33}$ Brussels, Bibl. Royale. MS II. 1639.
${ }^{34}$ For the date of the manuscript, see Smeyers (1999: 89).
${ }^{35}$ Sunderland (1959) argues that in Romanesque church plans in Burgundy the dimensions are based on the numbers three, four, five, seven and ten and the multiples thereof and suggests that these numbers had a symbolic significance.
${ }^{36}$ Suger of St. Denis (1979: 104-105): ‘Medium quippe duodecim columnae duodenorum Apostolorum exponentes numerum, secundario vero totidem alarum columnae Prophetarum numerum significantes, altum repente subrigebant aedificium, juxta Apstolorum spiritualiter aedificantem.'
${ }^{37}$ The whole idea of the spiritual church as represented in the material church is even more clearly expressed in a 12th-century ivory reliquary from the church of Sayn (now in the Musée Cinquantaire in Brussels), which shows a miniature basilica, made up of ivory plaques, with towers at the west and east ends. Under the arches of the aisles the figures of the twelve apostles are represented. On the front and back more saintly figures are shown under the arches.
${ }^{38}$ Abbot Simon's epitaph refers to him as a builder of the church: 'prudens sensu erexit ecclesiam [...]'.
${ }^{39}$ Wilson's idea that number symbolism may have been used in the choir of St.Rémi is compelling, but whether Pierre de Celle was the initiator of this use of number symbolism is no more than an assumption. That the abbot was the leading force behind the building campaign at St. Rémi, furthermore, as suggested by Otto von Simson, Anne Prache and others, is also no more than an assumption. As to the first assumption, there is no indication in the extensive writings of Pierre de Celle, including his surviving sermons, treatises and letters, of any interest in numerology. The second assumption, that Pierre de Celle was the main patron of the rebuilding of St. Rémi, is based on the following three brief references to the rebuilding of the abbey church, the most extensive one occurring in an undated letter to Prior Inganus of Lapley (letter 144): 'I am beginning to renew the chevet of our monastery, and with God's
help the work already begun looks well and promises noble auspices of the works to follow. I have not forgotten what you once said to me, as it were in rebuke, that I strove to do other works and not to care for the monastery. This word, even if it was said in passing, was not heard fleetingly. For I have laid out a thousand pounds so far, and at the same time have committed five hundred for a later stage, for the work on the monastery' (Pierre de Celle 2001, letter 144: 528-531). In letter 149 , addressed to the community of Grandmont between 1162 and 1181, he writes: 'We urge you in brotherhood, with humble supplication, to bestow the blessing of your holy fellowship on the benefactors of our church, and especially those who give their alms to the new work which we have begun to the honour and adornment of the house of God and of the blessed Remigius. For, wishing to renew the chevet of our monastery, with God's help we are putting out our hand to strong things' (Pierre de Celle 2001, letter 149: 546547). The last reference to the rebuilding of the abbey church is in a letter addressed to Richard of Salisbury, dated 1171: 'Meanwhile I wish neither to slumber nor to sleep but according to the counsel of the wise man to labour urgently at whatever I can. Hence it is that we have undertaken the construction of our noble church, both its façade and its body which was lacking a chevet in a fitting style, and, so we hope, with God's help we will finish it'. These three passages hardly qualify Pierre de Celle as a man with a passionate interest in architecture, nor do they form a sufficient basis for claiming that 'in Gothic architecture [... Pierre de Celle] sought and found the means to communicate his religious experience' (Von Simson 1974: 192).
${ }^{40}$ Unfortunately, Wilson gives no references for the passage concerning Rupert of Deutz. For Suger's Trinitarianism, see Suger of St. Denis (1979: 48-51): ‘Quod quidem gloriosum opus quantum divina manus in talibus operose protexerit, certum est etiam argumentum, quod in tribus annis et tribus mensibus totum illud magnificum opus, et in inferior cripta et in superior voltarum sublimate, tot arcuum et columnarum distinction variatum, etiam operturae integrum supplementum admiserit'. Wilson also mentions the west front of St. Denis as being described by Suger in Trintarian terms. I can find no such passage in Suger's writings.
${ }^{41}$ The northwest window is blind, due to the presence of a pre-existing tower in the angle between choir and north transept.
${ }^{42}$ For the dating of the various parts of the building, see Seymour (1939: 67).
${ }^{43}$ Kimpel and Suckale (1985: 125-136): ‘Das Querhaus hat keine Emporen [...] Deshalb konnte man im Querhaus auch den Aufriss verändern, konnte das Triforium verlegen. Funktionale Gründe hierfür vermögen wir nicht zu nennen, nur künstlerische. Das ist bemerkenswert genug [...] Die Kunsthistoriker haben sich immer daran gestört, dass im Querhaus von Noyon das Triforium nach unten gezogen ist, während die den Emporen entsprechenden hohen Fenster darüber
angebracht sind. Man hat dies als tastendes Experimentieren mit dem Aufriss gedeutet, als typisches Anzeichen für die noch nicht zu letzter Vollkommenheit gelangte "Frühgotik". Deshalb ist dieses Querhaus auch kaum bekannt und nur selten abgebildet. Wenn man jedoch die kunsthistorische Mythenbilding über die "Klassizität" des dreiteiligen Aufrisses hinter sich lässt und unbefangen die Bauten betrachtet, so wird man das Querhaus von Noyon eine der glänzendsten Architekturkompositionen der Gotik nennen müssen'.
${ }^{44}$ There are at least six versions of the Office for the Translation. All of these texts relate to the life of St. Thomas and his martyrdom, describe the actual events of the translation and interpret them within a scriptural and prophetic context, see Slocum (2004: 239-240).
${ }^{45}$ The passage is taken from Zacharias 3:9.
${ }^{46}$ Slocum (2004: 250, 274): 'Istis vero septem diebus martis non sine presagio hec gesta sunt; ut de huius septem intelligatur quod dicit dominus per prophetam, "Super lapidem unum septem oculi sunt"' (Office of the Translation), and (2004: 307-308): ‘We have now spoken of seven Tuesdays, beloved ones, which happened not without foreknowledge, as he who experienced battle and conflict on Tuesdays, brought back a glorious triumph over the conquered enemies on Tuesdays. Concerning those seven events, it can be understood as the Lord said through his prophet, "Above one stone there are seven eyes'" (Sarum Breviary Lesson 5) and 'Seven eyes are those seven events that occurred on Tuesdays, according to the orderly arrangement of the Holy Spirit. As a result of these Tuesdays it is possible to understand those things that are recounted in Revelation, the fact that John saw the Lamb that appeared to have been slain, having seven horns and seven eyes' (Sarum Breviary Lesson 6).
${ }^{47}$ Durandus, Rationale, cited by Bandmann (1981: 61): 'Quaecumque in ecclesiasticis officiis ac rebus in ornamentis consistunt, divinis plena sunt, signis atque mysteriis, ac singula sunt caelesti dulcedine redundantia: si tamen diligentem abeant inspectorem, qui noverit "mel de petra sugere oleumque de durissimo saxo".'

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How to cite this article: den Hartog, E 2014 1, 2, 3, 6: Early Gothic Architecture and Perfect Numbers. Architectural Histories, 2(1): 17, pp. 1-17, DOI: http://dx.doi.org/10.5334/ah.bu

Published: 20 June 2014
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