# Leonardo da Vinci: The Proportions of the Drawings of Sacred Buildings in Ms. B, Institut de France 

Francesco P. Di Teodoro*


#### Abstract

Previous studies of proportion in Leonardo's work have focused on the drawings of human anatomy and horses, and on the Sforza and Trivulzio equestrian monuments. Rarely has the interest of scholars concentrated on architectural proportions (Pedretti 1978; Schofield 1991); more frequently scholars have tried to 'ridurre in proportione' the plans or to arrange the sketches by Leonardo into real dimensions (for example, Guillaume and Kübacher (1987)). The purpose of this article is to understand what proportional schemes form the basis of Leonardo's architectural drawings and how the plans, elevations and architectural members are proportionally related.


## 1. Introduction

With the publication of the first anthology of Leonardo da Vinci, edited by Jean Paul Richter (Richter 1883; Pedretti/ Richter 1977; Di Teodoro 1992) the heterogeneous corpus of writings by Leonardo, scattered among various manuscripts and loose sheets, first became systematically catalogued. In the seventh chapter of this anthology ('On the Proportions and on the Movements of the Human Figure') are collected notes relating to the proportions of the human body. Since this publication, studies of proportions in the work of Leonardo have proceeded along the lines of Richter's thematic groupings, focusing especially on the anatomical drawings of humans (head, face, foot, hand, arms, legs, whole body) and of horses (preparatory for the Sforza and Trivulzio equestrian monuments). Rarely have such studies been concerned with Leonardo's architectural drawings.

Inspired by the opportune appearances of the first three editions of Architectural Principles in the Age of Humanism by Rudolf Wittkower $(1949,1952,1962)$ and The Theory of Proportion in Architecture by Peter Hugh Scholfield (1958), all of which contain references to Leonardo's architectural drawings as parts of broader discussions of architectural proportions (Fig. 1), ${ }^{1}$ Carlo Pedretti (1962: 130-136) was the first to examine an architectural drawing by Leonardo da Vinci with the primary purpose of intensively studying its proportions (Fig. 2). ${ }^{2}$ This drawing is the well-known perspective sketch of a sacred building, in the top right margin of f. 238 v , preserved in the Gallerie dell'Accademia in Venice and dating to 1515 (Pedretti 1978: 254). Pedretti's study went so far as to deduce from an external perspective view a precise floor plan. Pedretti's example has not been followed.

[^0]A systematic study of the architectural drawings of Leonardo was undertaken by Jean Guillaume (1987: 207-286), on the occasion of an exhibition in Montreal on Leonardo as engineer and architect. This study was undertaken from the point of view of typological groupings, based on attempted planimetric reconstructions from the drawings, similar to what Arnaldo Bruschi (1969: 175-178) had previously done for fifteenth-century centralized structures. For this exhibit, the premises of which were formally laid out in an essay published the following year, Guillaume and Krista de Jonge (1988) examined the same central-plan temple that had aroused the interest of Scholfield, Codex Ashburnham 2037, f. 5v = Ms. B, f. 95v (Scholfield 1958: 52 and Plate 7). From a plan measuring $90 \times 73 \mathrm{~mm}$ and an exterior perspective view measuring 73 x 66 mm , they deduced a complete project (plan, elevation and section). Described down to the most minute details of the orders, ornaments, openings, roofs and structure; and translated into a wooden model of great size, it was one of the highlights of the exhibition in Canada. In my view, however, the model was not only far from expressing the intentions of Leonardo, whose drawing gives no indications of the interior, or of the arrangement of the floor plan, but was inconsistent with the proportions of the two diagrams from which it originated (pertaining to the elevation). Indeed, most of the architectural drawings of Leonardo do not lend themselves to being studied as if they pertained to real-life projects.

## 2. The Drawings of Sacred Buildings in Ms. B

As a first approach to the study of the proportions in the architectural drawings of Leonardo, I have focused only on the sacred buildings sketched in Ms. B. ${ }^{3}$ This choice is due to the following facts:

1. These drawings are all similar subjects (mostly variations on the theme of the central plan or of composites thereof); ${ }^{4}$


Figure 1: Reprinted from Scholfield, The Theory of Proportion in Architecture (Cambridge, 1958), 141. At right: Leonardo, Ms B, f. 95v, detail. Note similarities between Scholfield's lower right star diagram shown here, and the star pattern formed by construction lines in the center Leonardo's Ms B floor plan.
2) they are all proposed according to the same mode of representation (usually a plan accompanied by a perspective elevation);
3) they were executed around the same time.

The subject of sacred buildings is perhaps the most striking of all those treated in Ms. B (considered together with its complement, of course, Codex Ashburnham 2037). Available for consideration are eighty drawings, ${ }^{5}$ ranging from plans, diagrammatic schemes, perspective elevations, sections and details, concentrated in folded sheets 1 to 4,6 , and 10 of the codex. ${ }^{6}$ There are only eight basilicas represented, in six plans and two perspective views (ff. $11 \mathrm{v}, 24 \mathrm{v}, 35 \mathrm{v}, 52 \mathrm{r}, 57 \mathrm{r})$. Among them are the plan of the basilica of Santo Spirito in Florence (f. 11v) and two plans of the Holy Sepulchre of Milan (f. 52r).
The drawings in question that depict central-plan churches are seventy-two in number. Among them are the planimetric schemes of Brunelleschi's Rotonda degli Angeli (f. 11v) and Santa Maria in Pertica, in Pavia (f. 55 r ). In some cases these drawings consist of elementary graphic lines: ff. 15 r (three plans in the right margin), 21r (two plans with corresponding elevations), 52r (a basilica plan at lower left), 93v (two small plans at upper right and left), and others showing more complex schemes.
A limited group of figures consists of more elaborate drawings of larger size. They are drawn in pen and shaded for enhanced three-dimensional rendering that makes them stand out from the sheets. This group is composed of pairs, each consisting of a plan and a perspective exterior view, and each relating to a particular building (ff. $17 \mathrm{v}-18 \mathrm{r}, 18 \mathrm{v}-19 \mathrm{r}, 21 \mathrm{r}, 21 \mathrm{v}, 22 \mathrm{r}, 24 \mathrm{r}, 25 \mathrm{v}, 39 \mathrm{v}, 52 \mathrm{r}, 93 \mathrm{v}, 94 \mathrm{r}$, $95 \mathrm{v}) .{ }^{7}$ There are nineteen pairs, among them only two basilicas. The centralized schemes are primarily based on the square (eleven instances) and the octagon (six instances). The plans are almost always drawn at right and the elevations at left ${ }^{8}$ - a sign that the latter preceded


Figure 2: Reprinted from Pedretti, A Chronology of Leonardo da Vinci's Architectural Studies after 1500 (Geneva, 1962), 131-135.
the former in execution (ff. 17v-18r, 18v-19r, 22r, 24r, 25v, $39 \mathrm{v}, 52 \mathrm{r}$, and the pair at left on 93 v of the five overall). ${ }^{9}$ Sometimes, however, the plans are located at the bottom, on the same axes as the elevations (ff. 21r, drawing in the lower right; 93 v , four of the five pairs present on the sheet; 94r, 95 v ), ${ }^{10}$ or shifted a bit to the left (f. 21r, drawing at upper left).

The plans are always in the form of geometrical 'wire frame' diagrams. Only at a later time did Leonardo partially or fully 'dress' them with wall thicknesses: thicknesses that not only generate inconsistencies in the relationships between solids and voids, in the structural plausibility of wall thicknesses, and in the connections between the parts, but also change the proportional relationships. ${ }^{11}$ The rapid abandonment of this exercise, which runs contrary to the original graphic procedure, betrays the differences between ideation, rapid graphic notation, and complete building design. The clarity of the drawing in the temple schemes in Ms. B - often completely or partially made freehand - in effect de-emphasizes the materiality of the buildings thus projected.

Elevations, on the other hand, are in the forms of perspective views: curves, lateral surfaces of hollow solids, and shells that suggest that the volumes and shapes, regardless of how all the internal elements of the plan can be developed structurally to become articulated above ground. The exterior is only one possible form through which the plan can evolve in space, as a kind of vertical extrusion. The structure of the interior, in fact, is never indicated in section (except in the case of f .4 r of the Codex Ashburnham 2037 = Ms. B, f. 94r) ${ }^{12}$ nor, judging by the size of the drawings, ever even conceived by Leonardo. On the other hand, when trying to correlate the exterior with the interior, Leonardo resorts to broken perspectives (for example, Codex Atlanticus, f. 205v-a), views showing internal details, or transparent views. Examples of this type - and I consider here not only the sacred buildings


Figure 3: At left and upper center: Leonardo, Ms B, f. 34v and graphic elaboration thereof; in upper right, Ms B, f. 55r, detail.

- are also present in Ms. B (ff. 12r, 12v, 15r, 16v, 19r, 21v, $36 r$, $36 \mathrm{v}, 37 \mathrm{r}, 37 \mathrm{v}, 38 \mathrm{v}$, 39r, 47r, 94r, 95r).

With the composite plans, the perspective views always look toward the apse, i.e., the part that corresponds with the main central space of the building, which is the most complex and articulated. Clearly Leonardo is here reflecting on the basilical form in light of the central scheme.

Some plans, if not all, study the various possibilities of a given form, and are akin to 'ludo geometrico' - 'geometric play' (Heydenreich 1974: 41). These plans are characterized by the iteration of components and the decomposition of the initial figure, as can be found in numerous other drawings by Leonardo (for example: Codex Atlanticus, f. 307v). In particular, the multiplication of chapels and circular or semicircular apsidals surrounding the buildings (especially of the central schemes) recalls the example of Brunelleschi's Santo Spirito and how such arcuated perimetric structures stiffen the construction. Later, in Ms. A, f. 51r (1492), Leonardo would express the structural value of this procedure in very clear terms in his well-known formula: ‘L'archo [...] farà il suo ofitio $p(e r)$ qualu(n)que v(er)so si stia, o rovescio o a diacere o ritto'. In the period 1505 to 1510 , furthermore, in the Hammer Codex (ff. 12v, 23v, $25 r$ ), the mechanism that causes domes to crack would be compared to that of sonagli ('bells'), or evanescent and fragile air bubbles on the surface of water. In this case, the cracking would be contrasted with a circle of smaller sonagli for buttressing' (Hammer Codex, f. 12v), thus proposing again, therefore, the scheme of a central-plan church, domed and surrounded with chapels (Chastel 1987; Di Teodoro 1991).

The multiple graphic solutions that Leonardo proposes in the pages of Ms. B are for his own use, just like the notes that sometimes accompany them and that integrate or explain some characteristics or novelties of the planimetric and spatial invention.

## 3. Method of Investigation

This study begins with the geometrical interpretation of the plans. It examines the proportional relationships that connect plans with elevations in a few significant examples. This interpretative approach only considers


Figure 4: At left and upper center: Leonardo, Ms B, f. 35r and graphic elaboration thereof; in upper right: Ms B, f. 55r, detail.
the intrinsic characteristics of the drawings and does not introduce external elements into the analysis that would attempt to transform drawings that are, and remain, only schemes and general studies into something possible and concrete, or feasible. The investigation does take the context provided by the content of the codex into account, for such context reveals a certain manner of working on Leonardo's part, through chains of association. Indeed, it is no coincidence that Ms. B contains no fewer than fourteen geometrical constructions involving plane figures, especially triangles and regular polygons (ff. 12v, 13r, 13v, 14r, 17r, 27v, 28r, 29r, 40r), and that four of them involve or include the octagon (ff. $12 \mathrm{v}, 17 \mathrm{r}, 40 \mathrm{r})$, drawn beginning from a given square, or from the circumscribed circumference thereof, or from an indicated side. Numerous studies of buildings in the codex also insist on the octagon and on the ribbed dome surmounted by a lantern with an octagonal plan, demonstrating, moreover, the extent to which Leonardo continues to reflect on the dome of Santa Maria del Fiore, both with regard to its form and to the geometry associated with its structural qualities. ${ }^{13}$
The first operation performed on the designs concerns, therefore, their geometrical bases (in particular of the plans), in order to verify their outlines, especially when the drawings were executed freehand. Such is the case, for instance, in ff. 34 v and 35 r , where two drawings, apparently geometrical exercises, refer to the plan of 'Santa Maria in P(er)ticha da Pavia' drawn on f. 55r of the same codex (Figs. 3, 4). ${ }^{14}$

Relating the proportions of the plan to those of the perspective elevation, even when freehand drawings are concerned, is permitted especially when important sides of the building in perspective are presented parallel to the perspective frame. This, for instance, is the case in the sheets of Ms. B considered here. The perspective views are almost cavalier axonometrics; the vanishing point is always to the left, and the horizon is very high, so as to establish a view from above that reveals very well the concatenated assemblage of volumes. I have refrained from relating plans and elevations when the latter were too small to furnish sufficiently detailed information. Significant results


Figure 5: At left and lower right: Leonardo, Ms B, f. 52r and graphic elaboration thereof; at upper right: Ms B, f. 57r, detail.


Figure 6: Leonardo, Ms B, f. 24r: graphic elaboration of the two drawings in the upper half of the sheet.
are obtained, instead, when the plan and the perspective elevation are on almost the same scale.

An example in which it seems preferable not to relate plan and elevation is the compositional scheme of f . $52 r$ (Fig. 5), where the plan, at lower right and located beneath the schematic plan of the 'teatro da p(re)dicare', ${ }^{15}$ measures $122 \times 80 \mathrm{~mm}$; and the perspective elevation, a small drawing in the upper left margin, measures only 27 x 22 mm (Firpo 1963: 58-59; Guillaume 1987: 236, 237 fig. 259; Schofield 1991: 140, 148). ${ }^{16}$ The plan reflects on the organization of the church of the Holy Sepulcher in Milan (which appears in two drawings on f .57 r of Ms. B: a plan of the church and one of the crypt) ${ }^{17}$ - a medieval scheme of the quincunx ${ }^{18}$ with two added bays - which Carlo Pedretti (1978: 23-24) has related also to the first ideas for the new cathedral of Pavia (together with the drawings of ff. 24 r and 55 r of the same codex).
The following examples concern one basilical scheme and three centralized structures in which both the square and the octagon appear, and the planimetry of which is more complex than might at first seem. ${ }^{19}$

### 3.1. Ms. B, f. $24 r$

It is possible to establish a precise relationship between the plan and the elevation of the composite building of $f$. 24r (cf. Firpo 1963: 56-57; Guillaume 1987: 228, 229 fig.

236; Schofield 1991: 138, 140, 148, 150, 151, 156), drawn, again, as a suggestion of the Holy Sepulchre in Milan (Fig. 6). It comes with a note, written underneath the perspective view:

Questo edifitio è abitato di sop(r)a e di sotto; di sop(r)a si va p(er) li campanili e vassi su p(er) lo piano dove sono fondati i 4 tiburi, e detto piano à $j^{\circ}$ parapecto $\operatorname{din}(n)$ ançi, e di detti tiburi nessuno ne riessie in chiesa, ançi sono sep(er)rati i(n) tucto. ('This building is habitable both below and above; the way up is accessed by the campanili, and in going up one uses the level where the drums of the four domes rest, and this level has a parapet in front, and none of these domes communicate with the church, but they are quite separate' (Richter 1883: II, 53)).

Leonardo conceives of the four 'tiburi', the small domed structures corresponding with the corners of the rhomboid body of the church, which touch and connect with four of the eight sides of the drum, with no visual relation to the interior, as buttresses, similar to the 'tribune morte' of Brunelleschi (Barbi and Di Teodoro 1983; Di Teodoro 2011). The octagon is regular and, assuming the module that corresponds with the side of the bay of the aisles (a 1 x 1 square), one finds that the nave has bays equaling 1 x $\sqrt{2}$ (the nave width is, therefore, equal to the diagonal of each bay of the aisles) and that the sides of the octagon therefore measure $\sqrt{ } 2$ in length.

Comparing the scale of the elevation with that of the plan, it is possible to verify how the elevation is constructed geometrically from the plan, and more particularly from only certain elements of the central square. The height of the first cornice surrounding the building on the top of the semi-cylindrical body of the apse and the lateral tribunes equals two modules; the height of the crowning cornice (the support for the balustrades), by contrast, equals the width of the nave: it is therefore equivalent to $\sqrt{ } 2$. The remaining portion of the square outline of the building, excluding the apse, in the end determines the height of the drum..$^{20}$ The dome rises, therefore, from a hypothetical cube. It has been possible to determine the dimensions of the elevation because Leonardo has drawn it to the left of the plan, in direct relation to its central part. Yet it is also clear that Leonardo intended to further study and refine this part. The longitudinal body, in fact, is articulated very little in the perspectival view, and lacks its lateral portico.

### 3.2. Ms. B, f. 39v

The study of f. 39v (Firpo 1963: 42; Guillaume 1987: 230 and fig. 240; Schofield 1991: 138, 150, 151), the planimetric characteristics of which are reconstructed starting from its octagonal satellite in the upper left of the sheet (Fig. 7), concerns a centralized building consisting of an octagon inscribed in a square and surrounded by eight evenly spaced elements of octagonal plan that, in elevation, correspond with an alternation of bell towers and protruding


Figure 7: At left and above right: Leonardo, Ms B, f. 39v and graphic elaboration thereof; lower right: Ms B, f. 25 v , detail.
niches. The towers (with a variety of examples of terminations) are connected to the central structure by means of narrow passages that make the towers communicable with the small exedral spaces that abut the secondary sides of the internal octagon. The central octagonal body has a pyramidal roof (like the Florentine Baptistery of San Giovanni) while forms of the roofs of the eight protrusions, rather than being polygonal, are suggested by the various shapes of the circumferences. Underneath the perspectival view Leonardo writes: 'senp(r)e uno edifitio vole essere ispichato dintorno a volere dimostrare la sua vera forma' ('a building should always be detached on all sides so that its form may be seen' (Richter 1883: 36)).

Notwithstanding the incongruities between the plan and elevation, and the imperfections in the perspective view (the exedra on the right occupies the same space as the bell tower, and is against a side of the drum, while that on the left is entirely separated and distant from the drum), in this case, too, the elevation can be identified on the basis of the plan. The lower level - corresponding to the parallelepiped on which the drum rests - has a height equal to the sum of twice the apotema (radius of the octagon measured to the center of any side) of one of the octagonal satellites and the side of the square space between two of those satellites. It is also equal to twice the side of the square space between two of those satellites (yellow line, in Figure 7). The same is true for the height of the drum (red line in Figure 7). The two portions of the building thus seem to stand in a $1: 1$ relationship. The planimetric scheme of f. 39v, through Leonardo's elaborations on f. 25v (Firpo 1963: 39, 41; Guillaume 1987: 230 and fig. 241; Schofield 1991: 139, 148, 150) (lower right in Figure 7), gives way to two ulterior, distinct schemes proposed in the pairs of drawings on ff. 19r-18v (example 3.3) and $18 \mathrm{r}-17 \mathrm{v}$ (example 3.4).

### 3.3. Ms. B, ff. 19r-18v

In the first pair, ff. 19r-18v (Fig. 8), as, in fact, in the second, the plan is on the sheet to the right (f. 19r), and the perspectival elevation, the one to the left (f. 18v) (Firpo 1963:


Figure 8: Leonardo, Ms B, details of ff. 19r (plan) and 18 v (perspectival elevation): graphic elaborations.

38-39; Guillaume 1987: 229 and fig. 238). Note that, in conformance with the left-handed writing, Leonardo fills his notebooks 'from the bottom up', that is to say from the right to the left. Thus the elevation was drawn after the plan was sketched: this practice supports the hypothesis proposed here of an orthography that is generated from the plan by means of simple geometry, based on particular graphic elements in the planimetry. The perspective view of f .18 v is accompanied by the note: 'A nessuna chiesa sta bene vedere tecti, a(n)çi, sia rapianato e p(er) chanali l'acqua dissie[n]da ai chondotti fatti nel fregio' ('It never looks well to see the roofs of a church; they should rather be flat and the water should run off by gutters made in the frieze' (Richter 1883: 37)).

The plan, an inscribed cross with an octagonal nucleus, has been drawn by Leonardo both with the help of tools (lines are clearly drawn with the ruler and the four half circles of the perimeter are executed with a compass) and in freehand (the niches of the four-lobed chapels and the four circumferences along the diagonals of the square), expanding an earlier, entirely freehand drawing, still clearly legible in the upper left quadrant of the plan.

Geometrically the plan is composed of a square divided into nine minor squares, some of which are in turn further divided into sixteen smaller squares. The, four-lobed chapels thus each have a nucleus of sixteen squares and are articulated around the major square. Each of these chapels penetrates this major square by one-quarter of the chapel's width, about to touch the central octagon. The circumference, only fully sketched out in the part that corresponds with the external surface of the chapels, has a diameter that can be defined according to its tangentiality with the internal octagon. This octagon, as can be verified directly from measurements, is constructed on the basis of the square whose side forms the golden section $(\Phi)$ with the side of the perimeter square. This construction can also be found to be reflected in the one Leonardo adopts on f .26 v of Codex K2 (= Cod. K, f. $74 \mathrm{v}) .{ }^{21}$ Once the octagon and the four major circles are constructed it is possible to trace also the minor ones,


Figure 9: Leonardo, Ms B, details of ff. 18r (plan) and 17 v (perspectival elevation): graphic elaborations.
each of a diameter equal to half of the major ones and tangential to the external square.
The comparison of the geometry of the plan and the perspective elevation suggests that the height of the side of the main parallelepiped is exactly half the square that circumscribes the plan; that the height of the drum stands in a golden section relationship with the height of one of the sides of the parallelepiped body; and, finally, that the dome will be as high as the parallelepiped itself.

### 3.4. Ms. B, ff. 18r-17v

The golden section occurs also in the proportioning of the central structure whose plan is on the upper right hand side of f .18 r and whose perspective elevation is found below on f. 17v (Firpo 1963: 46-47; Guillaume 1987: 226 and fig. 227; Schofield 1991: 138, 150, 151), both identified with the letter 'A' (Fig. 9). The elevation is accompanied by the note 'Questo edifitio anchora starebbe bene a ffarlo da la linia abcd in su' ('This edifice would also produce a good effect if only the part above the linea a b, c d, were executed' (Richter 1883: 46)), referring to the plan ABCD that divides the drawing in two according to the crown of circular domed chapels flanking the high drum, equally domed, of the octagonal nucleus. This drawing refers back to the solution already envisioned on $\mathrm{f} .25 \mathrm{v}-$ upper section - without, however, the little apses around the chapels.
The construction of the plan first involves the definition of the circular chapels on the basis of the tangent with the circumference circumscribed about the central octagon. As can be deduced approximately from the drawing, joining the center of the octagon with the extreme opposites of the diameters of the circumferences of the chapels should equal the double radius. On the same drawing by Leonardo, it can be seen that the diameter of the octagon has a golden section relationship with the side of the circumscribing square. In this way the satellite circumference tangent to the upper left angle is directly defined. Drawing the circumference passing through the center of this satellite, therefore, and within the circumscribing square, will define the midpoints of the other seven chapels of the crown.

The elevation is constructed so that the compact parallelepiped base will be as high as the plan is framed by one side of the square and the line (parallel to it) passing through the centers of two sides of the octagon. The total height of the bases and drums of the radial chapels equals half of the plan (a half side of the square). The drum corresponding with the octagonal nucleus, finally, goes from the line that in the plan indicated the top of the parallelepiped body (which coincides with the plane indicated by Leonardo as ABCD ) and that which, on the opposite side, passes through the intersection of the ideal circumference which contains the midpoint of the radial chapels with the extensions of the two opposing sides of the octagon.

In this proportioning of the elevations, more than in the other elevations considered hitherto, emerges a true and proper proportional scheme. In fact, as becomes apparent from the last scheme, at lower right in Figure 9, the geometry of the plan, with its remarkable lines and its projections, seems able to define many of the fundamental elements of the elevation.

## 4. Conclusions

Most of Leonardo's architectural drawings still await examination with regard to proportion. Still, the exploration conducted on the sheets of Ms. B - which is only slightly antecedent to Leonardo's studies of the problem of the Tiburio of Milan Cathedral, in which he thus comes into contact with the realities of construction ${ }^{22}$ - has brought to light some modalities that Leonardo puts to use (valid at least for the drawings of this codex from Paris):

- Leonardo rapidly derives the elevations from the plans;
this method is facilitated either by the fact that the plans are drawn before the elevation, or by the reciprocal position of the two drawings;
- in the simplest examples the elevations are defined starting from a particular element of the plan by means of progressions (for the pair at the top of f. 21r the progression is $1,2,4,8$ );
in the more complex cases the lines passing through quite particular portions of the plan define the heights and the cornices of the exterior (ff. 24r, 39v, $19 \mathrm{r}-18 \mathrm{v}, 18 \mathrm{r}-17 \mathrm{v}$ );
in some cases the golden section intervenes either in the definition of the plan (ff. $19 \mathrm{r}-18 \mathrm{v}, 18 \mathrm{r}-17 \mathrm{v}$ ) or in that of the elevations (ff. 18r-17v).

The recourse to the golden section is not odd because Leonardo, who applies a simple graphic method, seems to have been familiar with it since his Florentine years and at least since the beginning of the 1470s.

## Author's Notes

The preparation of Figures 6,8 and $\mathbf{9}$ is credited to Filippo Camerota. The study of Leonardo's drawings, Ms. B, ff. 24r (Fig. 6), 19r-18v (Fig. 8), and 18r-17v (Fig. 9) is credited to the collaboration between the writer and Filippo Camerota. Ursula Zich has redrawn some of my graphics on the computer. The transcriptions of Leonardo's various
steps follow conservative criteria; I simply distinguished $u$ from $v$ in the single grapheme $u / v$. I have retained the form ç. The 'a' verb (third person singular, present indicative of the verb 'to have') has been rendered with an accent (à). Abbreviations are enclosed in parentheses; brackets enclose additions.

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## Notes

${ }^{1}$ To Scholfield (1958: 52, 139-141) is owed one of the first contributions pertaining to the proportions of the plan in the temple in f .95 v of Ms. B of Leonardo da Vinci, as part of a broader study of the octagon star scheme (with reference to number theory and the number $\vartheta$ of the Pell series; see Figure 2).
${ }^{2}$ Pedretti (1962: 178) refers to Wittkower (1952), the second edition of Architectural Principles in the Age of Humanism, and makes explicit reference (1962: 133 n. 63) to the work of Scholfield.
${ }^{3}$ For the drawings of Ms. B and the themes explored here see, in particular, Heydenreich (1929); Maltese (1954: 333-358, especially 343-346 and plates CXXX-CXLV); Heydenreich (1974); Firpo (1963); and Schofield (1991: 137-143). For some issues in the codex (bridges, pilings, pile drivers, barns, fireplaces), see Di Teodoro (1985; 1993; 2009 and [forthcoming]).
${ }^{4}$ For the central plan, in addition to the fundamental Wittkower (1949) and Lotz (1964 and its Italian translation, 1989: 43-47); see also Adorni (2002).
${ }^{5}$ In arriving at this number I have also considered structures that are not 'churches' per se, but that contain a centralized or basilical plan: a plan and perspective section of the 'Pavilion of the Duchess's Garden' ('Padiglione del giardino della duchessa'; f. 12r) and the 'preaching theater' ('teatro da p(re)dicare'; ff. 52r, 55r, 95r).
${ }^{6}$ The drawings are distributed in the ten folded sheets that originally formed the codex as follows (the folded sheet 10 constitutes the present Codex Ashburnham 2037): folded sheets 1 (5), 2 (13), 3 (24), 4 (8), 6 (12), 10 (18).
${ }^{7}$ In ff. $17 \mathrm{v}-18 \mathrm{r}$ and $18 \mathrm{v}-19 \mathrm{r}$ the plans are on the right sheet and the perspective views are on the left; $f$. 21 r bears two distinct pairs of drawings; in f. 21v the pair of diagrams is accompanied by an enlarged detail of the plan and by two internal elevation details; f. 25 v bears two pairs of drawings; f. 93v contains five pairs of diagrams, while in f .94 r the plan and perspective elevation are accompanied by a partial section. The other sheets ( $22 \mathrm{r}, 24 \mathrm{r}, 39 \mathrm{v}, 52 \mathrm{r}, 95 \mathrm{v}$ ) each contain individual pairs.
${ }^{8}$ Exceptions are the drawings of f .21 v , in which the elevation is flanked by a fully drawn plan (at left) and a partial but larger plan (at right).
${ }^{9}$ This practice also applies to the drawing pair found on f. 12r (the Duchess's Pavilion, or, il padiglione della duchessa) and that of f .27 r (ribbed vault patterns of interlaced arches).
${ }^{10}$ This norm holds also for the pair in f . 15 r (with a wood truss roof over a square plan).
${ }^{11}$ This problem also occurs in the graphic transposition of steps in Alberti's De re aedificatoria. Consider, for example, the proportions of the three planimetric typologies for basilicas (simple, with causidica, and double-doored) that do not take into account the wall thickness (Alberti, De re aedificatoria VII.14).
${ }^{12}$ It is a church with two levels set on a square base with an inscribed octagon. Only a section view of a portion of it is shown. See also note 7.
${ }^{13}$ For the proportioning of the cupola of Santa Maria del Fiore and the geometric positioning of the intermediate ribs (sixteen in total, located in pairs, side by side, among the eight angle ribs), see D'Agata, Di Teodoro and Mancini (1977) and Di Teodoro (2012).
${ }^{14}$ In the first (Fig. 3), f. 34v, the plan is organized around a central octagonal chamber and the distribution of the ambulatory columns is derived from the intersection of circles having radii equal to half the side of the octagon, alternately planting the compass point in the centers of the sides and at the angles. The reference to an accretive method characterizes three sides of the figure. In fact, to the circles with centers at the angles are added those concentric circles with radii equal to the sides. Contrary to what appears in Leonardo's freehand drawing, these larger circles do not pass through the intersections of the smaller ones. In the second drawing (Fig. 4), f. 35r, instead, the columns are identified by the intersections of concentric circles, alternately with radius equal to half the side of the octagon, and with radius equal to the line joining the centerlines of two adjacent sides. The latter circles define the outer perimeter of a centralized building of eight lobes buttressed (or articulated with pilasters), as shown in the left portion of the drawing, where one wall thickness is called out in correspondence with four arches. In the first case the columns are arranged with a greater regularity (they are two different oncenter intercolumniations) than the second (where there are three types of intercolumniations).
${ }^{15}$ Leonardo also examines the 'teatro da predicare' (preaching theater) in f. 55r of Ms. B ('theaters for saying mass') and f . 95 r of the same codex ('place for preaching'). The design of f . 55 r , on the other hand, executed in pen, shows a centralized scheme (a square with four exedrae - one drawn in pencil - and ambulatories), but hints at, with the extension of some pencil lines, a possible longitudinal body - a solution that is taken up and developed in f .35 v . As it is well-known, the 'preaching theater' consists of an amphitheater-like structure (or simply a centralized scheme), tiered, and provided with a high central pulpit. It is plausible to assume that in conceiving of such an architecture Leonardo was influenced by 'parlagio', a term used in Florence to refer to an ancient public amphitheater, home, according to local historians, of the 'parliament'. Villani (1990, 1: 56) writes: 'And in that room was commanded that they should go in the villa of Camarti by the river Arno, and build there a
parlatorio in order to be able to make the parliament, and leave a memorial: this building in our vernacular we have called Parlagio. And it was built round and with vaulted roof in some ways quite marvelous, with a public square in the middle. And then they began by degrees to build it all around. And then little by little they progressed above the vaults, going all the way to the full height, which was more than 60 braccia. It had two doors, and in this the people gathered to conduct the parliament. And in progressive order the people seated themselves: in the upper side nobles, and then descending in accordance with the nobility; and so it was that everyone in the parliament saw one another face to face. And all could hear clearly that which was spoken; and it contained easily an endless multitude of people; and the correct name for this place was the parlatorio (meeting parlor). This was destroyed in the time of Totile, but still today we find the foundations and part of the vaults near the church of San Simone in Florence, extending all the way to the start of Piazza Santa Croce; and part of the Peruzzi palaces are built on its foundations. And the street called Anguillaia, which goes to the church of Santa Croce, goes almost to the middle of that Parlagio.' The Ms. B also bears other Florentine remembrance notes: in f. 11v Leonardo drew plans (incomplete) of the Rotonda degli Angeli and the Basilica of Santo Spirito.
${ }^{16}$ The planimetric scheme of f. 52r derives from a modular grid in which the basic module (M) corresponds to a bay of the aisles and the tri-portico that wraps around the longitudinal main body. The nave is equal to modules, and the octagonal domed space, the center of the triconch organism based on the square, is defined by constructing inside the larger square (of side equal to 6 M ) a smaller square, the corners of which fall on the centers of the sides of the larger square. The irregular sides of the octagon are equal to 2 and to $\sqrt{ } 2$. The sides of the two squares are between them are in the ratio 2 : $\sqrt{2}$. The building, excluding the three exedrae, is formed of the sum of two squares.
${ }^{17}$ The drawings of f .57 r bear, in fact, the annotation, 'A è Santo Sepulcro di Milano di sop(r)a. B è la sua parte socto tera' ('A is the upper church of Santo Sepulcro of Milan. B is the part underground'). Cf. Guillaume (1987: 234) and Schofield (1991: 135). But see also Windsor, RL $12609 v$ and the Codex Atlanticus, f. 42v-c.
${ }^{18}$ Reflections on the quincunx scheme are in f .3 v of the Codex Ashburnham 2037 (= Ms. B, f. 93r).
${ }^{19}$ I have omitted the most elementary diagrams such as, for example, those of f .21 r : in the first pair (top left), the octagonal plan has four rectangular side chapels, each half a side deep (2: 1 ) and twice as high (2: 1 ). The octagonal drum on which stands a hemispherical ribbed dome is also twice as high as the side of the octagon. Overall, the structure is equal to four times the side of this polygon. In the second pair (lower right) the central octagon opens to four square chapels the side of which is equal to that of the octagon (1: 1). These chapels develop in height according to a cube surmounted by an octagonal drum surmounted
by a cupola, the edge of which is half that of the cube (2: 1). Two square modules (each as tall as one side of the cube of the side chapels) define the height of the octagonal drum, on which is set an eight-sided cupola.
${ }^{20}$ The discrepancy between the transverse dimension of the proportional scheme (in blue in Figure 6) and the 'ghost' drawing of Leonardo depends on the fact that the part of the façade to the right of the apse was drawn visibly narrower than the symmetrically opposing part.
${ }^{21}$ I refer here to the well-known graphical method for the construction of the golden rectangle from the square whose side becomes a proportional mean. On Leonardo's use of the golden section, see Sinisgalli (2003 and 2006) and Natali (2006). Camerota [forthcoming] demonstrates the use of the golden section in the 'Adoration of the Magi' (c. 1482) and in the determination of the dimensions of the panel and the position of the vanishing point in the central axis of the 'Annunciation' (c. 1472).
${ }^{22}$ For the Tiburio and the proportional questions that they involve, see in particular Ferrari da Passano and Brivio (1967); Guillaume (1987); Schofield (1989); and Di Teodoro (1989) and (2001). In my short essay of 1989, I showed how Francesco di Giorgio, indicating a height of 28 braccia for the height of the Tiburio (in reconnecting it with previous geometric schemes; Schofield 1989: fig. 1), was referring to the theory of the perfect number. It is worth emphasizing again that the use of such numbers was topical in the 1480s and 1490s, after the publication of Euclid's Elementa with commentary by Giovanni Campano (Venice 1482). I would add that the number 28 (the same as the number of coffers in each ring of the dome of the Pantheon) had already determined the number of episodes of Giotto's 'Life of Saint Francis ' in Assisi upper church and the panels of the north doors of the Baptistery of Florence. In the sixteenth century, this number would establish (together with the number 6) the proportions of the Florentine Ponte Santa Trinita by Bartolomeo Ammannati (see Di Teodoro et al. 1981).

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[^0]:    * Politecnico di Torino and Centro Linceo Interdisciplinare 'B. Segre'—Accademia Nazionale dei Lincei, Italy francescopaolo.diteodoro@tin.it

