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EDITORIAL

Conclusion: Ten Principles for the Study of Proportional Systems in the History of Architecture

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The study of proportional systems today lacks a common framework for discussion. In this conclusion to the special collection of essays on this topic published in *Architectural Histories,* Matthew A. Cohen provides ten principles as a basis for future discussion on this complex topic.

Introduction

In his keynote address to the conference 'Proportional Systems in the History of Architecture', hosted by Leiden University, 17–19 March 2011, Howard Burns posited that the study of proportional systems today lacks 'ground rules'. In response, I present the following list of ten principles as a suggested framework for new discussion of this complex topic.[1]

Principle 1: The word 'proportion' signifies two unrelated meanings

As I note in the introduction to this special issue of *Architectural Histories*, 'Introduction: Two Kinds of Proportion', the subject of architectural proportion today is literally defined by ambiguity because the very word proportion simultaneously signifies two unrelated and in some ways opposite meanings. Proportion technically denotes a mathematical ratio, which I call 'proportion-as-ratio', but in common usage it also connotes a broader meaning that in 1723 Ephraim Chambers described as 'a Suitableness of parts, founded on the good Taste of the Architect', or, an aesthetic assessment that I call 'proportion-as-beauty' (Le Clerc 1723–1724, 1: 29).[2] Since the first meaning is quantitative and the second qualitative, neither can influence the other predictably and repeatedly, as the scientific method would require in order to prove a causal relationship between them. When historians use the word proportion without qualification, however, they invite their audiences to understand it as a fusion of the concepts of ratio and beauty, and thus as an implicit assumption that certain proportional ratios contribute beauty to architecture (see Principle 10). Thus, in my 'Introduction' to this special collection, I have proposed that the word proportion be broken down into its incongruent component meanings that I have termed 'proportion-as-ratio' and 'proportion-as-beauty', and that one of these meanings hereafter be specified whenever scholars use this word, either through the use of the preceding terms or in the context of the discussion.

Principle 2: Proportional systems must be described with verifiable measurements

In any study of proportional systems that refers to proportional ratios either found in existing buildings or in historical drawings, those proportions-as-ratio must be described accurately, as they really are, based on measurements recorded directly from the object. Those measurements, in turn, must be described in sufficient detail to enable the reader to go to the object and measure between the same points of measurement in order to verify them. When point cloud laser scans are cited, the method by which the data were acquired needs to be made sufficiently transparent to enable the reader to spot potential errors and understand the limitations of the survey. Also, when possible, the digital model or detailed images derived from it need to be made accessible to the reader in order to allow verification of all measurements extracted from it. Descriptions of proportions that are not supported by verifiable measurements can at best be considered subjective interpretations of appearances, which may have their own value but lack the foundation of scholarly documentation. Finally, verifiability requires not only that authors provide measurements, but that readers insist on them and learn how to evaluate them rigorously.

Principle 3: Proportional systems are never executed exactly as intended

Since irregularity is the normal condition of architecture, some degree of dimensional discrepancy between the proportions the architect intended and the proportions of the built object are unavoidable. Potential structural degradation and measurement error must be considered integral to this problem. Part of the architectural historian's job, therefore, is to acknowledge these discrepancies and estimate their extents based on internal evidence from the object. For example, my San Lorenzo survey reveals that all six columns in the western three bays (first phase) of the nave differ in height by no more than 1.5

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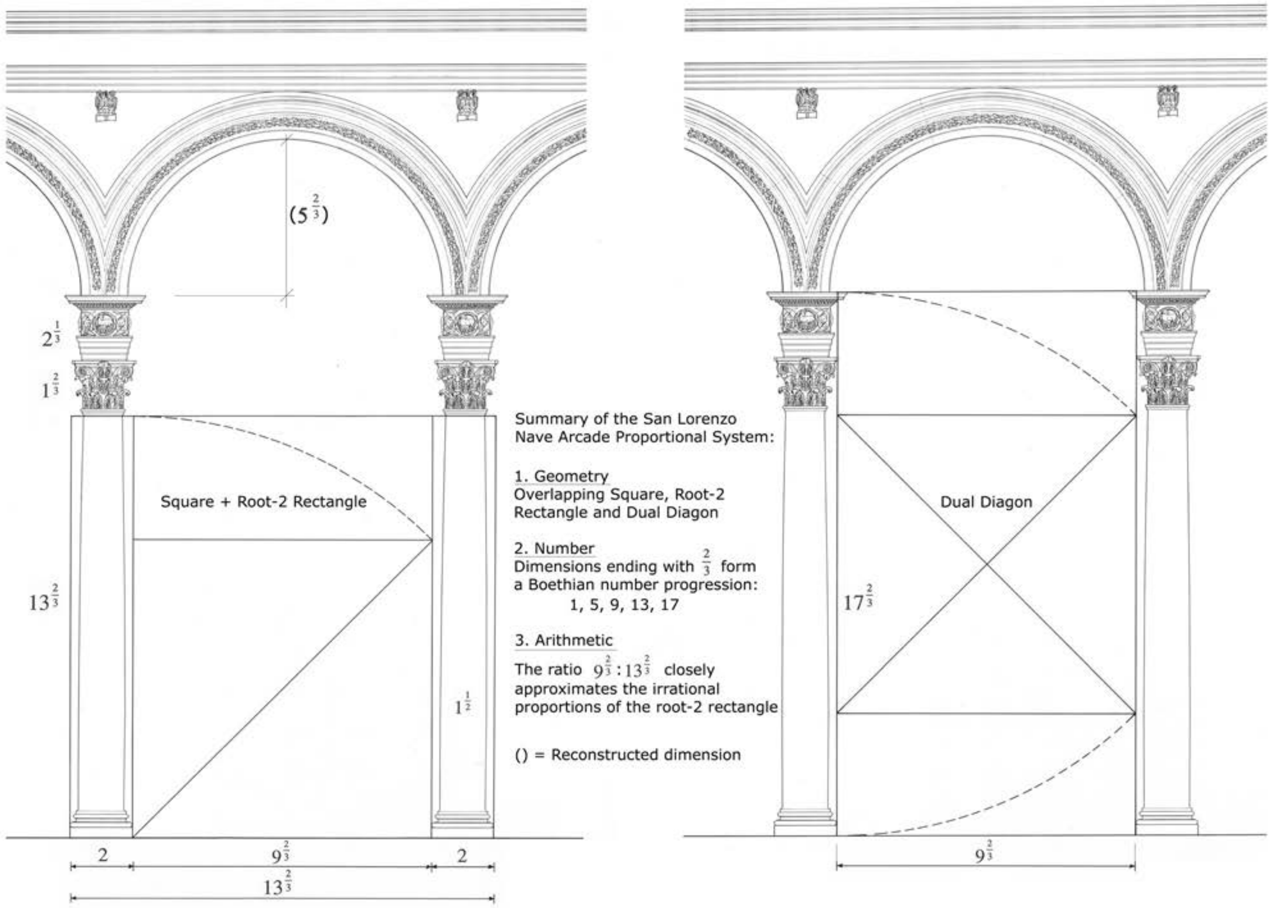

Fig. 1: Summary of the San Lorenzo nave arcade bay proportional system (spread between two drawings for clarity). At left is a geometrical overlay consisting of a square and a root-2 rectangle. At right is a geometrical overlay consisting of a rectangle called a dual diagon. Source: author.

centimeters from one to the next (**Fig. 1** shows the nave arcade bay design that is repeated eight times in each nave arcade).

If the masons could achieve this level of precision in duplicating column heights, I reasoned, they must have been able to execute the various width-to-height proportions that they intended between each pair of adjacent columns within the same level of precision. Therefore, in this part of the basilica I considered any proportional discrepancy (i.e., between an apparently intended proportion and the building measurements) that exceeded 1.5 centimeters to be abnormally large, meaning that either the architect did not intend that proportion after all, or that some extenuating historical circumstance that has to be identified caused the discrepancy. In other buildings, larger discrepancies appear to be normal. Such estimates need to be made case by case. Thus at San Lorenzo, based on the analysis described above, I determined that an 11- to 12-centimeter discrepancy between the column shaft heights and the height of a square + root-2 rectangle proportion that I propose was intended in each nave arcade bay (**Fig. 1**, left) must be the result of construction error. Note that the tops of the column shafts, where they meet the astragals, rise slightly above the top of the square + root-2 rectangle overlay in the accurate scale drawing in **Figure 1**, though they were probably intended to align perfectly with the top of this overlay (on this construction error, see Cohen 2013: 104–111; 2008: 33–37).

Principle 4: Proportional systems are executed in terms of local units of measure

Proportional systems may contain abstract proportional ratios, but once a building construction project begins, those ratios become expressed in terms of the local unit of measure. That unit may be specific to the municipality in which the construction takes place, or to the construction site itself.[3] I believe it is safe to say that every historic building (and perhaps many pre-historic ones) that has ever resulted from substantial expenditures of labor, resources and capital was built in accordance with such a unit, regulated by the supervising authority at the time and place of construction. Available evidence indicates that in historic buildings, measurements expressed in these units and their subdivisions often add layers of numerical information to proportional systems, in addition to any abstract ratios that the architect may have intended.

For example, the basic framework of the San Lorenzo nave arcade bay proportional system consists of three overlapping geometrical figures: the square, the root-2 rectangle, and the dual diagon (**Fig. 1**). These figures embody abstract ratios that we can describe mathematically as

Fig. 2: Ospedale degli Innocenti, Florence. Photo: author's archive.

1:1, 1:√2 and 1:2√2-1, respectively. The preceding numerals and symbols, however, are modern conventions of mathematical communication that are not represented as such in the early-15th-century San Lorenzo proportional system. Only when we convert the measurements from centimeters to Florentine *braccia* (1 *braccio* = 58.36 cm) do we see that the architect of this proportional system described these ratios in terms of numbers of *braccia*.[4] Significant numbers expressed as *braccia* in this proportional system include $1\frac{2}{3}$ *braccia*, $9\frac{2}{3}$ *braccia*, $13\frac{2}{3}$ *braccia* and $17\frac{2}{3}$ *braccia*. The architect used these numbers to form Boethian number progressions and numerical approximations of irrational ratios. He thus used numbers of *braccia* — the local unit of measure — to add numerical layers of meaning on top of the geometrical layers in this proportional system (**Fig. 1**; see Cohen 2013: 53–111; 2008: 19–37).

In his paper in this special collection and elsewhere, Stephen Murray notes that the two tallest Gothic cathedrals, Amiens and Beauvais, express the number 144, a New Testament dimension of the Celestial City, in their internal heights of the nave vaults above the floor in terms of their respective local units of measure. These numbers appear to be integrated into larger programs of iconographically significant numerical and geometrical relationships in these buildings. Murray's observation calls attention in a most poignant way to the important connection between units of measure, iconographical expression, and scale: In aspiring to this heavenly number symbolism, the Amiens builders had a somewhat easier structural task before them than their Beauvais brethren, whose soaring vaults collapsed in 1284, for the local Roman foot used in Amiens was slightly shorter than the royal foot used at Beauvais, and the physical vault height necessary to reach 144 local units of measure was consequently shorter by nearly six meters.[5] In light of this discussion we see that much theoretical commentary on proportional systems, such as Cesare Cesariano's plates that show the Cathedral of Milan's cross section with geometrical overlays (Cesariano 1521), and the treatises of Vitruvius (1st century BC; see Vitruvius 1914), Alberti (1485), and Vignola (1562) that describe proportional systems in terms of modules, neglect to acknowledge local units of measure, and thus provide an incomplete picture of how proportional systems worked in practice.[6]

Principle 5: Belief-based proportional systems often communicate through simultaneity[7]

We have seen that numbers expressed in terms of local units of measure can interact not only with individual dimensions, or with sets of dimensions associated with geometrical figures, to add layers of iconographical meaning to an architectural composition (see Principle 4). Additional layers of numerical meaning can be added through quantities of repetitious building components. Since numbers of units of measure (such as *braccia*) and numbers of building components (such as columns or bays) are fundamentally different from one another, their combinations in proportional systems can be profoundly interesting.

The portico of Brunelleschi's Ospedale degli Innocenti, for example, displays a total of ten columns. Indeed, it does so quite deliberately, for each end terminates with a full column standing next to the end wall, rather than an engaged column or a pilaster more typical of the period (**Fig. 2**). The distances between the columns measure precisely ten *braccia* on center.[8] Ten columns, ten *braccia*. You can see the columns and count them. You cannot see the *braccia*. You can only understand their quantities conceptually. This confluence of visible and invisible presences of the number 10 is consistent with *simultaneity* in medieval thought, perhaps a companion to the period's mystical conception of the visible world as but a reflection of a larger macrocosmic order (Crosby 1997: 46–47).

Examples of this visible/invisible proportional simultaneity abound in the history of architecture. In the Romanesque basilica of Santi Apostoli in Florence we find two nave arcades, each containing 6 freestanding columns spaced 6 *braccia* on center (**Fig. 3**).[9] Furthermore, the total

Fig. 3: Basilica of Santi Apostoli, Florence. Photo: author.

number of columns in this basilica, 12, simultaneously represents the number of apostles to whom the basilica is dedicated. The Cathedral of Milan floor plan is 16 bays long, and is composed of nave piers placed on a grid of 16 *braccia milanesi* on center (**Fig. 4**).[10] Brunelleschi's basilica of Santo Spirito has a nave 9 bays long, 3 transept and apse-like wings each 3 bays long (for a total of 9), and simultaneously, plinth to plinth distances of 9 *braccia* (Cohen 2013: 98–99, 105, 146).[11] Adding more simultaneity, the numbers 3 and 9 (the latter equivalent to 3 by 3) symbolize the Trinity, of which the Holy Spirit (Spirito Santo), to which the basilica is dedicated, is the third point. In the basilica of San Lorenzo, the 7 columns in each nave arcade correspond to the entablature block frieze reliefs symbolizing the 7-sealed Book of the Apocalypse (Cohen 2013: 149; Cohen, forthcoming).

Principle 6: Proportional systems constitute historical evidence

Proportional systems can facilitate insights into historical ways of thinking, as in my suggestion above that proportional simultaneity may have sometimes alluded to a larger macrocosmic order. They can also constitute historical evidence of a more specific nature. In the basilica of San Lorenzo, for example, I have identified an overall basilica proportional system that seems to call for deep, approximately square nave chapels rather than the present chapels that are only half as deep (**Figs. 5, 6**). This observation is consistent with Giuliano da Sangallo's drawing of the San Lorenzo floor plan that shows deep nave chapels (**Fig. 7**), and thus may indicate that Sangallo had inside knowledge that Brunelleschi intended just such chapels.[12] Whether or not this interpretation of this new proportional evidence is correct, it led me to search for precedents for this deep nave chapel scheme, and ultimately to find the late Gothic basilica of Santa Maria del Carmine in Pavia (**Fig. 8**). Upon visiting this building I not only found notable floor plan similarities with the basilica of San Lorenzo, as expected, but even more striking above-ground similarities between the Carmine and Brunelleschi's basilica of Santo Spirito (**Figs. 9** and **10**). These observations leave little doubt that Brunelleschi visited the Carmine and drew inspiration from it for his work, a significant and unexpected conclusion to which the use of proportional systems as historical evidence led me (Cohen 2013: 209–231; 2009).

Proportional systems, furthermore, provide sufficiently precise information to permit the development of provisional taxonomies of proportional systems, and from them, possible proportional system lineages from one building to the next. For example, in the nave arcade bays of the basilica of Santa Maria del Fiore, Francesco Talenti appears to have created a proportional system that includes overlapping rectangular proportions, plinth to plinth measurement, an accurate whole number approximation of the ratio 1:√2, and a strategic use of fractions (**Figs. 11** and **12**). Matteo Dolfini, in turn, appears to have used this proportional system as a source of proportional raw materials, including specific dimensions and proportions, for his design of the San

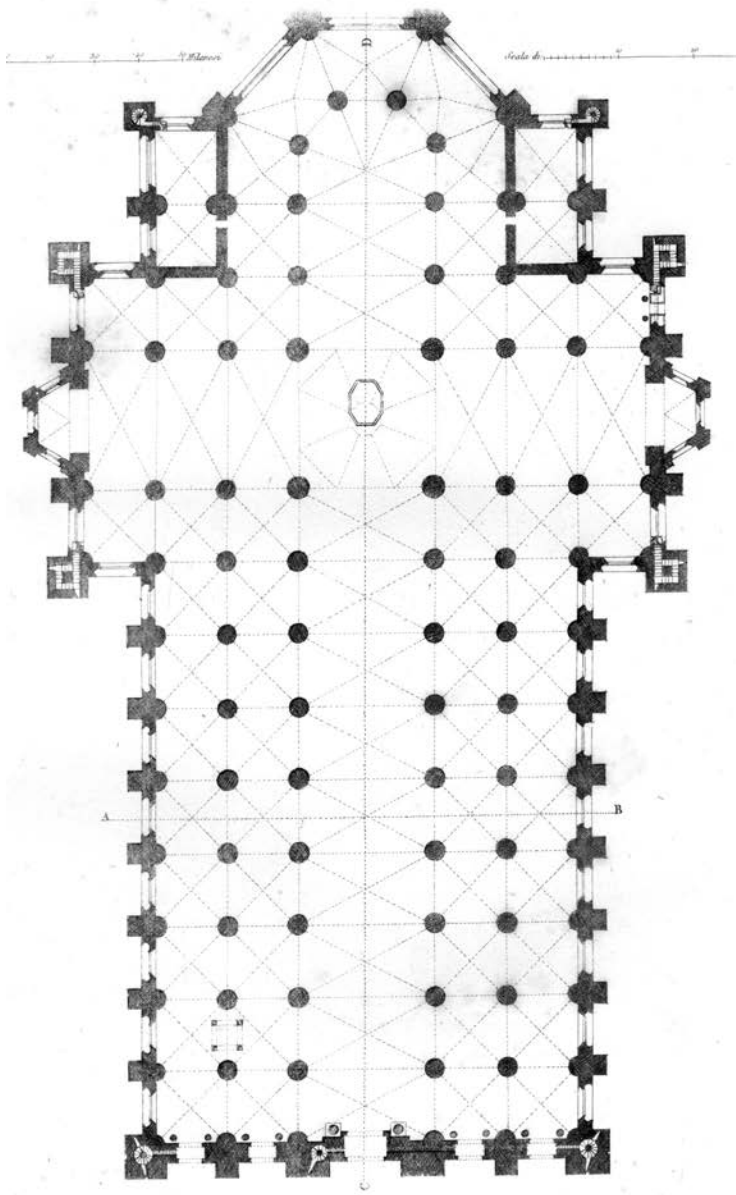

Fig. 4: Cathedral of Milan floor plan. From Gaetano Franchetti, *Storia e descrizione del duomo di Milano* (Milan: G.G. Destefanis, 1821), n.p.

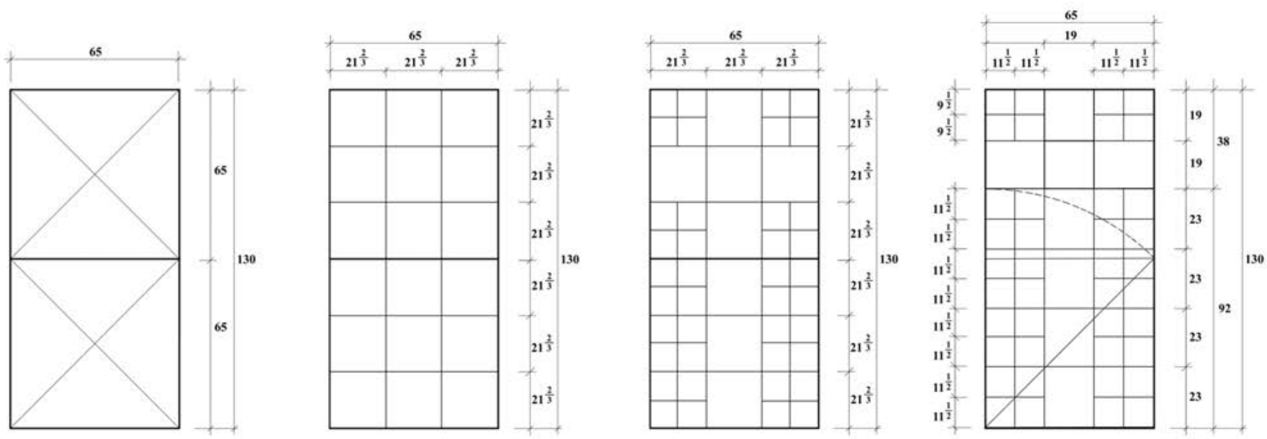

Fig. 5: Proposed diagrammatic derivation of the San Lorenzo overall basilica proportional system derivation, based on survey measurements. Root-2 rectangle overlay at far right indicates that a 65 x 92 *braccia* rectangle is a very accurate approximation of the mathematically irrational proportions of a root-2 rectangle. Source: author. *del duomo di Milano* (Milan: G.G. Destefanis, 1821), n.p.

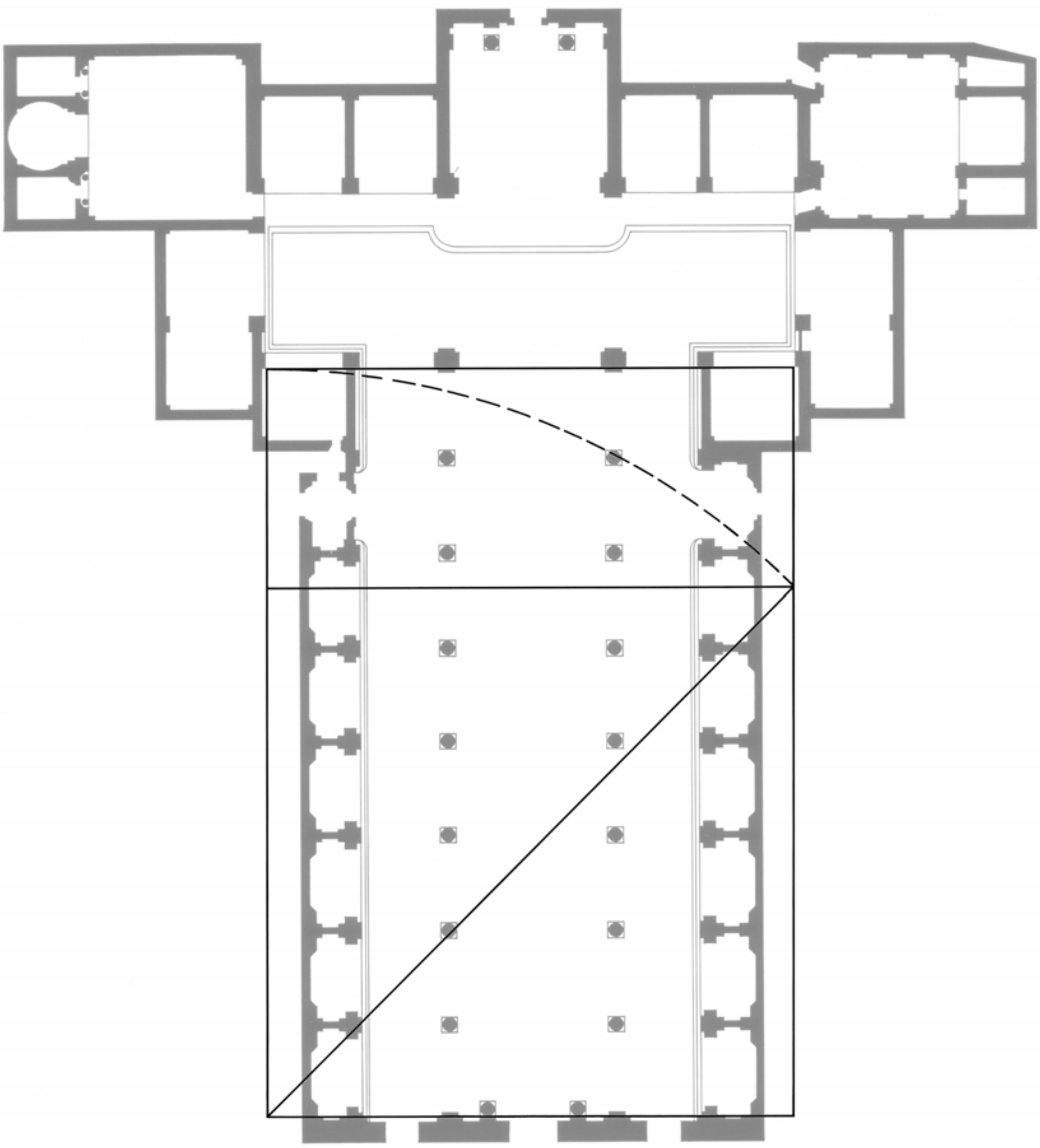

Fig. 6: Basilica of San Lorenzo floor plan with root-2 rectangle overlay, suggesting a possible original intention of deeper nave chapels.

Lorenzo nave arcade bay proportional system (Cohen 2013: 231–242; Cohen 2010). Dolfini died before he could execute his proportional system, but Brunelleschi appears to have adapted it to his redesign of the basilica we see today (**Fig. 1**). Brunelleschi, in turn, later used this proportional system as the starting point for his design of the proportional system of the Santo Spirito arcade bays, this time making more substantial changes to it (**Fig. 13**).[13] Thus we seem to have a Santa Maria del Fiore–San Lorenzo–Santo Spirito proportional system lineage that constitutes new historical evidence of previously unrecognized design influences between these three buildings.

Principle 7: Belief-based proportional systems have served no practical purposes

Elsewhere I have identified six intended purposes of belief-based proportional systems in the history of architecture, including structural, aesthetic, iconographical and perhaps philosophical categories (Cohen 2013: 25–35, and ch. 6).[14] Of these six purposes only structural stability can be considered practical, and belief-based proportional systems have never conferred it. The others are all subjective qualities that cannot even be said to have ever been present in any work of architecture except when someone has believed that they were.

Modularization, which might seem to be the most logical practical purpose a proportional system could have served, does not appear to have ever been used during the pre-engineering period to a sufficient extent to have saved substantial time and money in construction. The Roman practice of manufacturing column shafts to standard sizes once constituted the standardization of one building component and therefore did not constitute a proportional system, though it may have influenced proportional systems. Indeed, large-scale modularization for the purpose of economization appears to be a phenomenon of

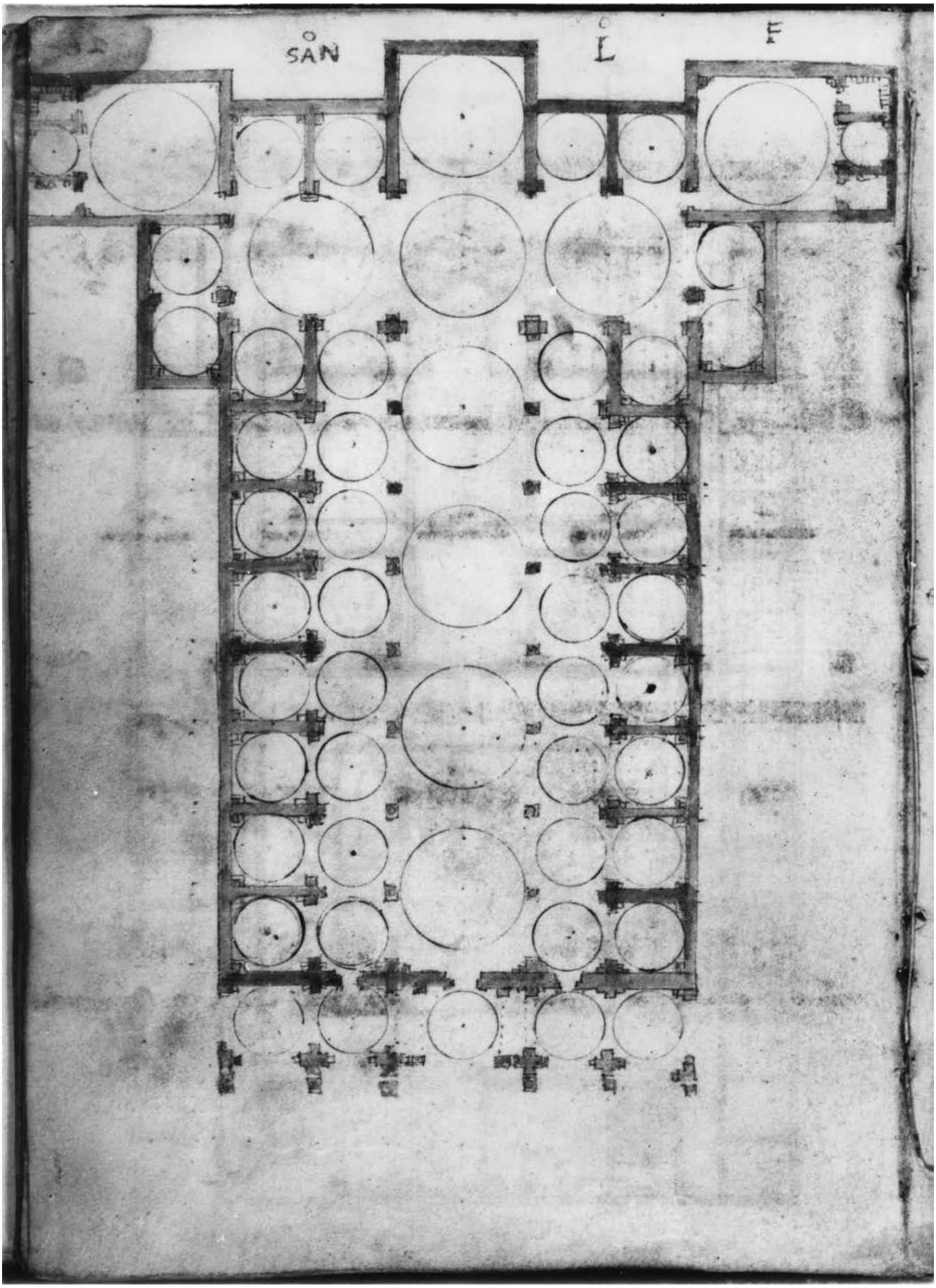

Fig. 7: Giuliano da Sangallo, Basilica of San Lorenzo floor plan sketch, 'Taccuino senese' (c. 1480). Source: Biblioteca Comunale, Siena.

mechanical production associated with the Industrial Revolution. Industrial modularization usually adopted the simplest possible units based on the prevailing unit of measure — either the meter or the Imperial foot — and had no need for other units, such as Le Corbusier's complicated Modulor, which attempted to impose an arbitrary new module, even if Le Corbusier developed it to be commensurable with the aforementioned prevailing units of measure. Consequently the Modulor was never widely adopted.[15]

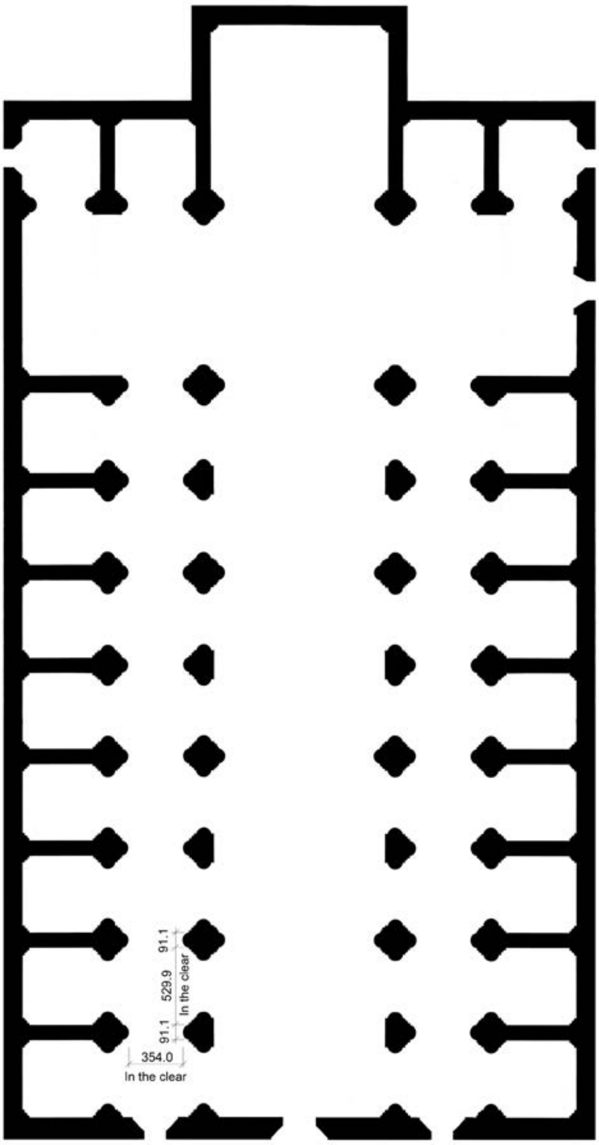

Fig. 8: Basilica of Santa Maria del Carmine, Pavia, floor plan. Source: author.

Principle 8: Proportional systems constitute design methods

Pre-engineering, belief-based proportional systems could facilitate an architect's creative process by generating forms and dimensional combinations from which he could make selections. These selections could then be added to the indistinct mix of external design influences and internal intuitions that fed the process. In the end these selections might equally likely have been incorporated into the proposed design, or discarded along the creative path leading to it. One need not agree with or even understand, for example, every line in the patterns of 'unfolding geometry' that Robert Bork (2011) proposes Gothic architects used in the designs of their complex works to be convinced that Gothic architecture would not have been Gothic but for the use of such a method.[16]

The beliefs of architects from the pre-engineering period that proportional systems were present in already-existing structures, furthermore, encouraged those architects to measure historical precedents both ancient and recent in order to discover those systems. Such observations led to many important design developments not necessarily related to proportional systems. Manetti's claim, for example, that in their youth Brunelleschi and Donatello went to Rome to measure ancient monuments in order to understand 'le loro proporzioni musicali' ('their musical proportions'), whether or not true, is more significant for its suggestion that the two friends studied ancient Roman monuments at all, rather than specifically their proportions (Manetti 1976: 66). The sketchbook of Villard de Honnecourt likewise betrays, in at least one proportional diagram sketch, an effort to discover the principles of French Gothic architectural precedents (De Honnecourt 1959: 92–93, pl. 41). Finally, Burns has aptly noted that during the Renaissance, proportional systems helped to 'slow down the design process', enabling the architect to ensure the design was exactly as he wanted it to be.[17]

Principle 9: Proportional systems contribute no aesthetic value to architecture

This principle, which has been treated in detail in my introductory essay to this special collection of *Architectural Histories*, is included in this list as a point of discussion, even though consensus as to whether or not it indeed deserves recognition as a principle may never be reached. It is my hope that the present collection of essays will inspire new and increasingly informed kinds of discussions of this question.[18]

Principle 10: Belief-based proportional systems communicate non-visual narrative content

The sole purpose of belief-based proportional systems has ever been to contribute meaning to architecture (see Principle 7). Such meaning can behave like a narrative, through a complex process that necessitates a longer explanation for this principle than the others. Unlike simple number symbolism, which is usually clearly visible and immediately comprehensible at least to educated observers (as in the 12 columns of the Santi Apostoli nave symbolizing the 12 apostles; see Principle 5), proportional systems reveal themselves over time, because by definition each consists of 'a set of geometrical, numerical or arithmetical correspondences'.[19] A single example of number symbolism can be part of such a 'set', but cannot constitute a proportional system in itself. The various parts of a proportional system reveal themselves one by one. Considering the simultaneity of tens in the Ospedale degli Innocenti discussed above, for example (see Principle 5), the observer is most likely to discover first that there are 10 columns. Only subsequently, after learning that the columns are spaced 10 *braccia* on center (either by measuring the portico or being told), might the observer realize that the number of columns is the same as the number of *braccia* between adjacent columns. The observer could also make these discoveries in the reverse order. Either way, one discovery constitutes prerequisite knowledge for the ultimate appreciation of the *simultaneity* of tens in this proportional system. The observer cannot, however, discover both sets of tens simultaneously.

Fig. 9: Basilica of Santa Maria del Carmine, Pavia, aisle view. Photo: author.

Fig. 10: Basilica of Santo Spirito, Florence, aisle view. Photo: author.

More complex proportional systems unfold more slowly, and sometimes in a more prescribed order. The San Lorenzo nave arcade bay proportional system, for example, took many years to unfold for me (Cohen 2013: 53–111; 2008: 19–37). First I discovered the overlapping geometrical figures of square, root-2 rectangle and dual diagon (**Fig. 1**). Next, by converting my measurements to Florentine *braccia*, I found that the latter two geometrical figures, which represent mathematically irrational ratios, are very accurately approximated by the ratios $9\frac{2}{3}:13\frac{2}{3}$ (which is equivalent to 29:41), and $9\frac{2}{3}:17\frac{2}{3}$ (which is equivalent to 29:53), respectively. Lastly I found that these numbers, together with others in the nave arcade bays that are also expressed in dimensions of *braccia* ending in the fraction $\frac{2}{3}$, form part of a Boethian number progression. The significance of each layer of this proportional system only became evident to me in relation to my understanding of the preceding layer. Today I require at least fifteen minutes to explain this proportional system to even the most expert listener, and I must present its various layers (consisting of geometry, number and arithmetic) one at a time, always beginning with the geometrical layer, since an understanding of this layer is prerequisite to an understanding of either of the others. Only after these layers have been unfolded sequentially can they be appreciated simultaneously, in the mind of the observer.

In light of this temporally unfolding quality, proportional systems can be understood as narratives, and perhaps plots, not only in the general senses of these terms as communicators of meaning, but in their specific literary senses as well. A narrative, according to Steven Cohan and Linda M. Shires, 'recounts a story, [or] a series of events in a temporal sequence' that leads to an 'outcome', or, 'closure' (Cohan and Shires 1988: 1, 65). This definition can be refined, these authors note, by referring to the novelist E. M. Forster's distinctions between the terms *story*, or 'a narrative that orders events temporally', and a *plot*, or 'a narrative that orders events causally as well as temporally' (Cohan and Shires 1988: 58).[20] Thus in a story, events take place in a particular order for numerous possible reasons, while in a plot they do so because earlier events influence later events. The British architect John Soane (1753–1837) recognized the fertile analogy between a similar notion of plot and architectural experience by comparing the latter to a theatrical performance. According to Caroline van Eck, Soane understood that 'looking at a building and watching a play are similar experiences [...] because both [engage] *temporal* arts'. The experiences of both theatre and architecture, she notes, 'unfold in the course of time', and in both, 'composition is built on the assumption that the viewer is able to follow the plot, or grasp the connections on which dramatic or architectural composition is based' (Van Eck 2007: 128).[21]

Proportional systems provide similar temporal experiences, though rather than based on *looking*, as in Soane's analogy, they are based on *comprehending*, accomplished through non-visual mental investigations of the geometrical, numerical and arithmetical relationships expressed

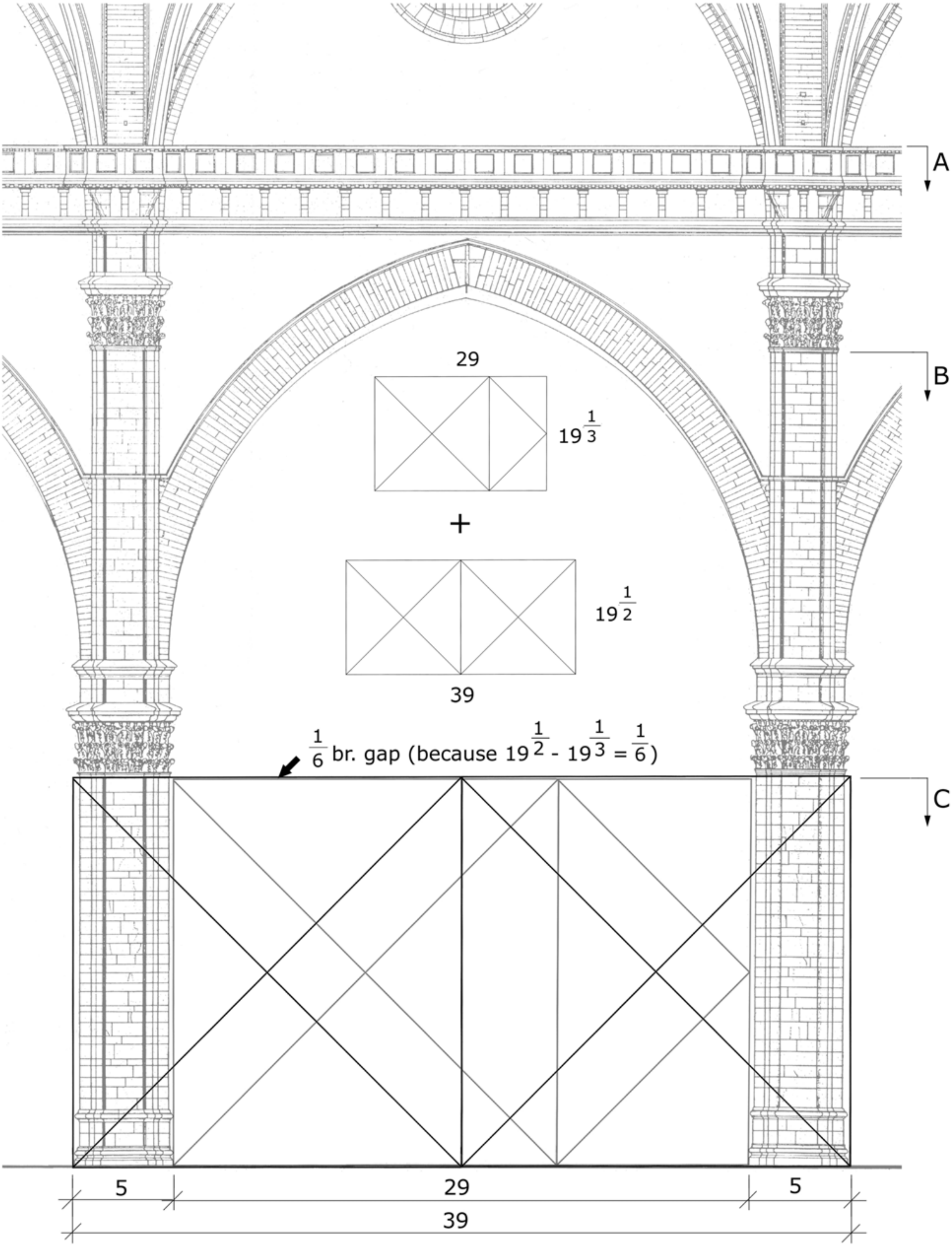

Fig. 11: Basilica of Santa Maria del Fiore, Florence, nave arcade bay proportional system, part 1. Source: author.

in the building measurements.[22] These proportional relationships could be just as laden with cultural associations as the physical objects that Soane uses to communicate the plot, such as portraits, sarcophagi and sculptures, even though these relationships are not physical. In the San Lorenzo proportional system, for example, elements that carry cultural associations include the root-2 rectangle, the number pairs that closely approximate this ratio (such as 29:41), and the Boethian number progression identified by the *braccio* dimensions that end in the fraction $\frac{2}{3}$. The root-2 rectangle attracted much attention in the 15th century because its incommensurability

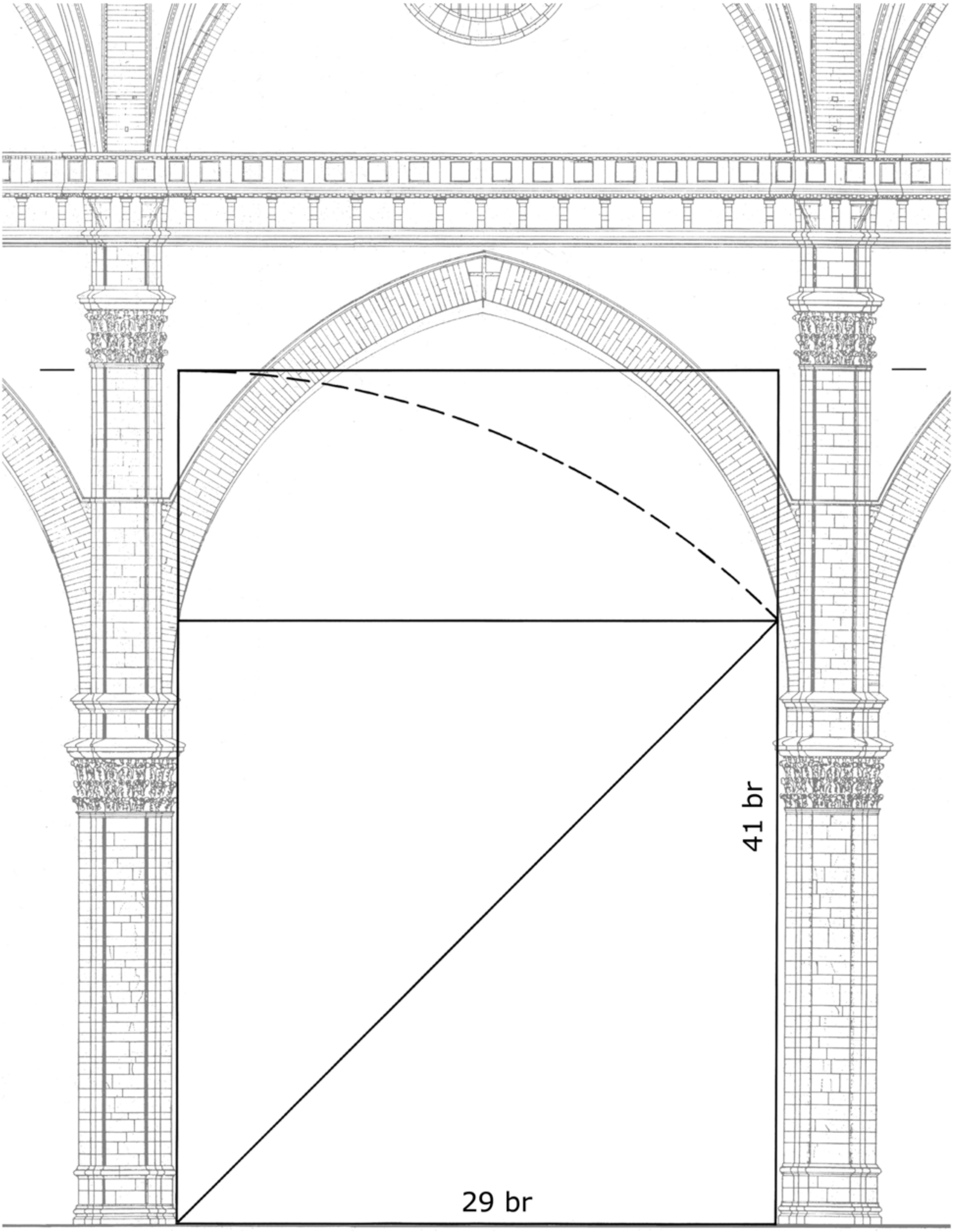

Fig. 12: Basilica of Santa Maria del Fiore, Florence, nave arcade bay proportional system, part 2. Source: author. system, part 1. Source: author.

presented a mathematical and philosophical dilemma. That dilemma, thinkers as early as Plato realized, could be resolved through accurate whole number approximations.[23] Finally, the number progressions I have termed Boethian carried particularly strong cultural associations because according to Boethius they formed the basis of arithmetic, the first step in the *quadrivium*, which led to the study of philosophy, the highest intellectual pursuit; and this formulation remained a basis for later categorizations of knowledge.

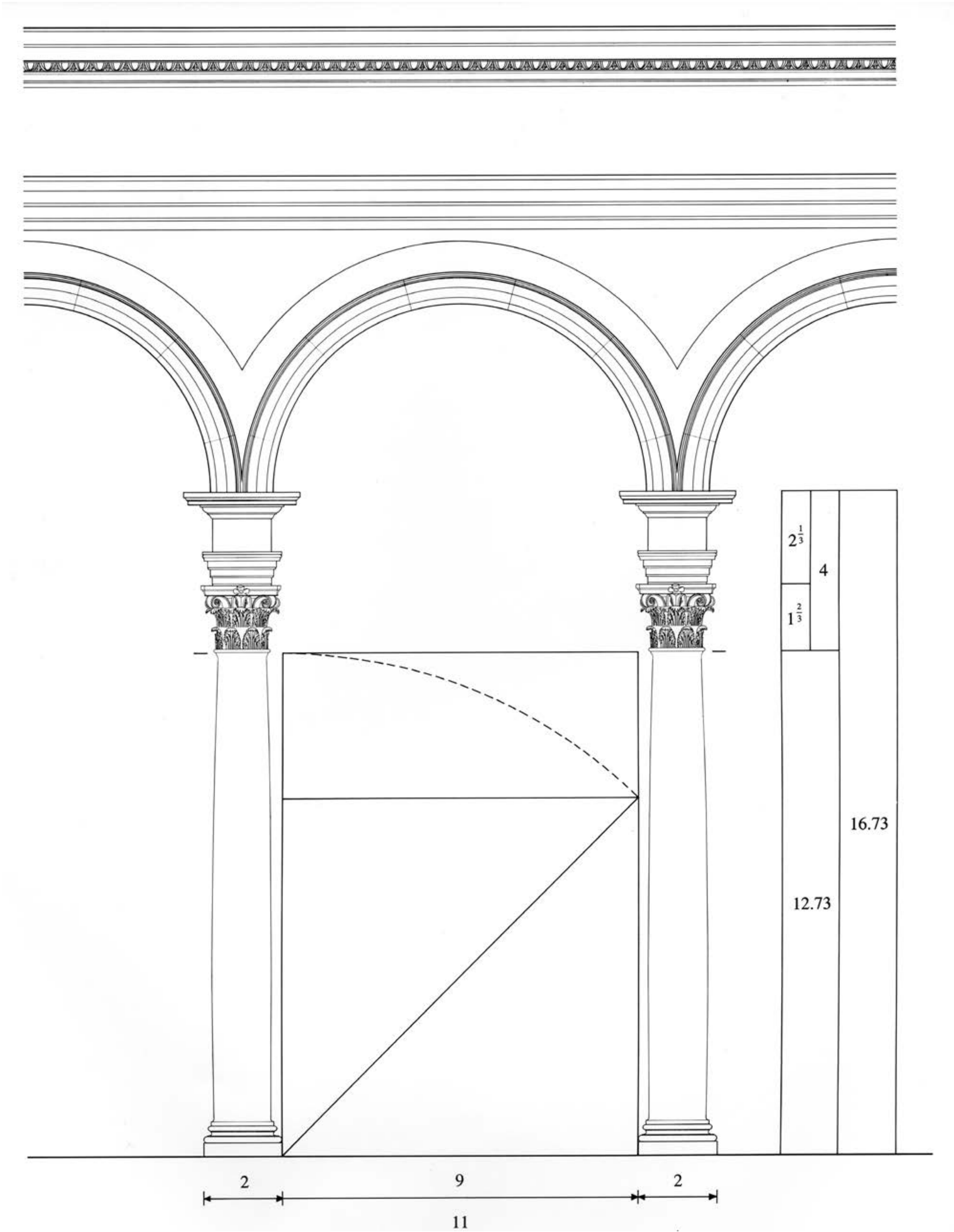

Fig. 13: Basilica of Santo Spirito, arcade bay proportional system. Source: author.

Conclusion

If proportional systems are to be understood as narratives or plots, what were their intended outcomes, and for whom were they intended? Proportional systems could have been intended as private communications from the architect to himself, as forms of private contemplation, or more likely, as semi-private communications from the architect and his patrons to those *cognoscenti* capable of deciphering them. In either case, they must have been important for the architects to have gone to so much trouble to craft them. As communications, furthermore, they may have been similar to prayers or incantations, for they may have facilitated contemplation of spiritual or metaphysical beliefs. Regardless of the exact character of any particular proportional system, the participants in such communications believed that proportional systems helped to bring about certain desirable states in architecture, such as structural stability, *ordine*, and perhaps a relationship with God and the macrocosm (Cohen, "Introduction: Two Kinds of Proportion" in this special collection). By imbuing lifeless stone with the agency of communication, proportional systems thus contributed to making architecture more than merely a functional art, but a deeply meaningful one. Deciphering these communications that reach us from across the centuries, in the absence of explicit documentary evidence, remains a central challenge for architectural historians. A big part of that challenge is to maintain historical objectivity, and to avoid any temptation to believe the messages.

Notes

1 The conference was organized by Matthew A. Cohen, Eelco Nagelsmit and Caroline van Eck. For more information on this conference, and for definitions of 'proportional system' and other terms, see Cohen, 'Introduction: Two Kinds of Proportion', in this special collection of essays.

2 Cf. Le Clerc (1714: 39): 'Par proportion, on n'entend pas ici un rapport de raison à la maniere des Geometres; mais une convenance de parties, fondée sur le bon goût de l'Architecte.' For a similar use of this term in Italian from the 16th century, see Van Eck's quotation of Cosimo Bartoli in 'The *Composto Ordinato*' in this special collection; and Cohen (2013: 21–22).

3 See Stephen Murray, 'Plotting Gothic: A Paradox', in this special collection; and the observation of Benevolo, Chieffi and Mezzetti (1968: 2–4), later confirmed by my own measurements and statistical analysis (Cohen 2013: 88), that the basilica of Santo Spirito in Florence was built according to a *braccio* slightly longer (58.67 cm) than the norm (58.36 cm).

4 For my conclusion that the church prior-architect Matteo Dolfini appears to have designed the San Lorenzo nave arcade bay proportional system but died before he could realize it, and that Brunelleschi inherited it from him, and modified it to varying degrees for use in both the basilicas of San Lorenzo and Santo Spirito, see Cohen (2013: 185–207), which substantially expands upon Cohen (2008: 41–44).

5 Murray furthermore notes that the 'spread of the west façade buttresses with the twelve minor prophets is 120 feet', a dimension that he suggests perhaps refers to the twelve foundations of the Celestial City and the 'twelve apostles of the Lamb' both of which are also mentioned in this passage of Revelations. The number 144 therefore may be part of a broader proportional system, rather than a single symbolic number; see Murray, 'Plotting Gothic: A Paradox', herein; and Murray (1996: 163). For units of measure used at Amiens, and Revelations 21:14–17 indicating the height of the wall of the Celestial City as 144 cubits, see Murray (1996: 159–160). Indeed, New York City's Freedom Tower, which recently topped out at a symbolic 1776 feet, would have required either a different symbolic number or a vastly increased height had the United States adopted the metric system, as it considered doing several decades ago.

6 These authors probably intentionally suppressed local units of measure to make their publications universally applicable.

7 For the term 'belief-based proportional system', see Cohen, 'Introduction: Two Kinds of Proportion', in this special collection.

8 The average portico bay width, on center, based on my measurements of the Ospedale recorded in June 2005, is 584.05 cm. I use 58.36 cm as the assumed length of the Florentine *braccio*. On the *braccio* see Cohen (2013: 8–89). Saalman (1993: 71) also notes an on-center bay width of ten *braccia* in this portico, but does not provide the survey measurements from which he derived these bay widths.

9 According to my measurements recorded in June 2005, the diameters of the bases of the column shafts, which Vitruvius recommends as a module, measure 59 cm. Since this dimension is very close to the value for the early 15th-century Florentine *braccio*, 58.36 cm, it raises the interesting possibility that the Florentine *braccio* was already in use during the 11th century, albeit with a slightly longer dimension than in later centuries. The on-center distance between the column shafts measures 353.6 cm, or, nearly exactly six *braccia* (dividing by either 59 cm or 58.36 cm). For the purposes of this discussion I have assumed that the basilica of SS Apostoli was built according to a *braccio* of 59 cm.

10 Although the last two bays at the apsidial end are angled and therefore do not measure sixteen *braccia* lengthwise, I am counting the length of the floor plan in bays to arrive at the number sixteen, not *braccia*. For evidence of the sixteen *braccia milanesi* grid of the Cathedral of Milan floor plan, see Stornaloco's letter and diagram in Frankl (1945: 53).

11 My interpretation of the significant dimensions in the Santo Spirito floor plan as being those measured plinth to plinth (i.e., 9 *braccia* between adjacent column plinths) is not necessarily in conflict with Giuliano da Sangallo's annotation to his Santo Spirito floor plan drawing with two on center measurements, 22 *braccia* and 11 *braccia*, see Da Sangallo (1984, 2: fol. 14r). Evidence suggests that during the late medieval and early Renaissance periods, dimensions that were originally conceived by their architects as measured plinth to plinth were in later documents reported on center, as in this drawing, perhaps for convenience. For a similar discrepancy between measurements reported in the documents pertaining to the nave arcades of the basilica of Santa Maria del Fiore, and the proportional system according to my interpretation and analysis, see Cohen (2010) and (2013: 232–242).

[12] One of the two floor plans of Brunelleschi's basilica of Santo Spirito drawn by Giuliano da Sangallo, furthermore, includes two notable differences with the present configuration, and surviving evidence indicates these differences indeed represent Brunelleschi's original intentions, perhaps as documented in his model for the basilica. The plan in question is drawn with four front entrances rather than the present three, and the semicircular chapels that surround the cruciform central space of the basilica are visible to the exterior, there being no flat enclosing walls as at present (Benevolo, Chieffi and Mezzetti 1968: 34–49; Da Sangallo 1984, 2: fol. 14r; Saalman 1993: 374–377; Sanpaolesi 1962: Fig. 54). A third difference, the continuation of the interior arcades across the interior facade, has no evidentiary support linking it to Brunelleschi apart from this drawing. Sangallo's drawing of the floor plan of the basilica of San Lorenzo (**Fig. 7**, herein) presents a more complex problem because it shows four differences with the present plan, at least one of which was surely not intended by Brunelleschi: numerous circles in the plan implausibly indicate all spaces throughout the basilica, not merely the crossing square and Old Sacristy, covered by domes. Other additions by da Sangallo, in addition to the deep, approximately square nave chapels, include an entrance portico, itself covered by five domes, and a second sacristy symmetrically opposite the Old Sacristy, drawn at a time when there is no clear evidence that such a sacristy had indeed yet been planned (Da Sangallo n.d.: fol. 21v). Some scholars think that if even one of these additional elements can be identified as out of conformance with Brunelleschi's intentions, then all of them should be assumed to be purely of Sangallo's invention. I am of the opinion, however, that Sangallo could have combined restoration of some of Brunelleschi's original intentions (the deep nave chapels, and conceivably the portico, probably not including the domes, and the second sacristy) with more fanciful elements of his own invention (such as the domes).

[13] See above, n 4.

[14] For the term 'belief-based proportional system', see Cohen, 'Introduction: Two Kinds of Proportion', in this special collection.

[15] To determine whether and to what extent the Modulor was used in 20th-century buildings, the buildings in question would have to be subjected to rigorous measurement-based analysis.

[16] See also Bork, 'Dynamic Unfolding and the Conventions of Procedure', in the present special collection.

[17] In personal correspondence with the author.

[18] For additional treatment of this question, see Cohen (2013).

[19] See Cohen, 'Introduction: Two Kinds of Proportion', in this special collection.

[20] Cf. Aristotle's definition of plot as 'the ordering of the particular actions' (Aristotle, *Poetics*, 1450a–1451a, as quoted in De Jong [2010: n 25]). For an alternative interpretation of the notion of plot in architecture see Murray, 'Plotting Gothic: A Paradox', in this special collection; and Murray (2011).

[21] For additional discussion of parallels between theater and architecture, see De Jong (2010: 334–351).

[22] 'Numerical' correspondences are the numerical qualities of integers as revealed, for example, in number progressions, while 'arithmetical' correspondences are relationships between numbers that are revealed through simple calculation (cf. Cohen 2013: acknowledgements page and 22).

[23] For an explanation of why the elements of the San Lorenzo proportional system are not physical, see Cohen, 'Introduction: Two Kinds of Proportion', in this special collection. On the philosophical dilemma presented by the root-2 rectangle, see Cohen (2013: 102–104).

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