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RESEARCH ARTICLE

Dynamic Unfolding and the Conventions of Procedure: Geometric Proportioning Strategies in Gothic Architectural Design

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This essay explores the proportioning strategies used by Gothic architects. It argues that Gothic design practice involved conventions of procedure, governing the dynamic unfolding of successive geometrical steps. Because this procedure proves difficult to capture in words, and because it produces forms with a qualitatively different kind of architectural order than the more familiar conventions of classical design, which govern the proportions of the final building rather than the logic of the steps used in creating it, Gothic design practice has been widely misunderstood since the Renaissance. Although some authors in the nineteenth and twentieth centuries attempted to sympathetically explain Gothic geometry, much of this work has been dismissed as unreliable, especially in the influential work of Konrad Hecht. This essay seeks to put the study of Gothic proportion onto a new and firmer foundation, by using computer-aided design software to analyze the geometry of carefully measured buildings and original design drawings. Examples under consideration include the parish church towers of Ulm and Freiburg, and the cross sections of the cathedrals of Reims, Prague, and Clermont-Ferrand, and of the Cistercian church at Altenberg.

Introduction

Discussion of proportion has a curiously vexed status in the literature on Gothic architecture. On the one hand, it is obvious to even the most casual observer that the proportions of Gothic buildings and their constituent parts, which are often very tall and slender, contribute significantly to their visual impact by suggesting upward movement and transcendence. On the other hand, though, it has proven difficult to explain exactly how these proportions arose in the design process. Indeed, the shockingly non-classical proportions of Gothic buildings famously led Renaissance writers like Vasari to conclude that this *maniera tedesca* was inherently wayward and disorderly.[1] In the five subsequent centuries, many more sympathetic authors have attempted to analyze and describe the logic of Gothic architectural proportions. However, while some valuable work has been done in this direction, the overall state of the field remains strikingly primitive even today. All too often such work has been flawed by imprecision, ambiguity, and wishful thinking. Many scrupulous scholars, therefore, have become skeptical about all such research, concluding that it reveals more about the pet theories and preoccupations of the researchers than it does about medieval design practice.

Fortunately, recent developments in the study of drawings, the surveying of buildings, and the use of computer-aided design (CAD) systems now allow the proportions of historic monuments to be studied with new rigor. It is finally becoming possible, therefore, to speak with reasonable certainty about the working methods of Gothic designers. To show this, the present essay presents two groups of CAD-based case studies: the first considers medieval drawings related to the design of the great spired towers at Ulm and Freiburg-im-Breisgau; the second considers the cross sections of the cathedrals of Reims, Clermont-Ferrand, and Prague, and of the Cistercian church at Altenberg. These case studies will demonstrate that Gothic design methods involved the dynamic unfolding of geometrical constructions.[2] This approach to design produced proportional relationships qualitatively different than those seen in the more static and module-based formal order of classicism. In a sense, therefore, Vasari was right to say that Gothic buildings lacked 'every familiar idea of order', although this comment says more about his own limitations than it does about the Gothic builders he sought to criticize.

The complex and procedurally based formal order of Gothic architecture, in fact, offers a highly sophisticated alternative to the classical tradition, one with real relevance for present-day architectural practice. Gothic buildings often exhibit patterns of self-similarity, in which details such as pinnacles echo the forms of larger elements such as spires, creating a rich resonance between microcosm and macrocosm. Analogous patterns are now seen in the mathematical objects known as fractals, and in the work of contemporary designers who use computer algorithms to develop complex and innovative formal systems of their own.[3] Geometrical analysis of Gothic design thus has the potential to enrich architectural practice in the

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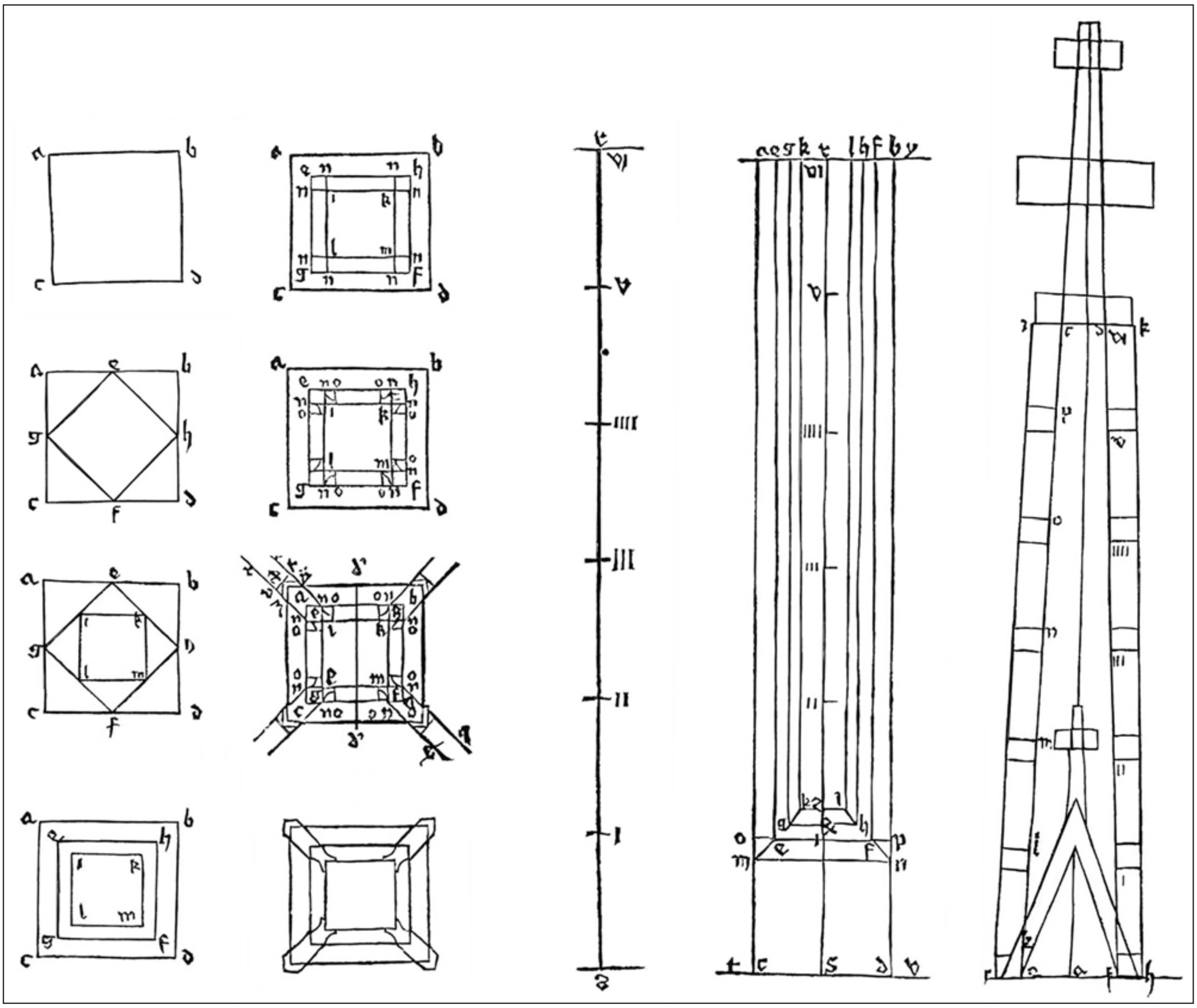

Fig. 1: Steps in the design of a pinnacle, from Matthäus Roriczer's 1486 *Buchlein von der Fialen Gerechtigkeit*, arranged by the author. Photo: Guido Pressler Verlag.

twenty-first century, in much the same way that formal and archaeological analysis of Gothic buildings enriched architecture in the nineteenth and early twentieth centuries. To see why the present analyses offer something new and valuable to the discussion of medieval design practice, it will be helpful to consider briefly some of the historiographical developments that have shaped scholarly understanding — and misunderstanding — of Gothic proportioning systems.

The problem of Gothic proportions, from Villard to Hecht

Gothic builders themselves left behind no very satisfying records of their methods. This fact is hardly surprising, since their training emphasized visual rather than verbal communication. The so-called portfolio of the thirteenth-century draftsman Villard de Honnecourt admittedly provides some scattered commentaries among its many drawings, but it certainly provides no sustained discussion of Gothic design practice.[4] Most surviving medieval documents of the building process are simply unillustrated construction ledgers, while most surviving design drawings have no textual glosses. Gothic builders were certainly able to convey their techniques effectively from master to apprentice, as the continuity of their traditions over more than four centuries demontrates. However, Gothic design conventions governed the rules of the process more than the shape of the final product, which meant that the spatial relationships between building components varied far more widely in Gothic than in classical architecture.[5] This, in turn, meant that precision could only be achieved by explicit demonstration and description, rather than by allusion to venerated prototypes. The procedural dynamics of Gothic creativity, therefore, could not easily and concisely be translated into words that would be understandable or satisfying to an educated layman. In this important sense, the geometrical logic of Gothic design was 'unspeakable' (Bork 2011a).

In the decades around 1500, nevertheless, several German late-Gothic authors attempted to explain their design methods in short pamphlets.[6] Matthäus Roriczer's 1486 *Buchlein von der Fialen Gerechtigkeit*, for example, illustrates the successive steps in the design of a pinnacle (**Fig. 1**).

Roriczer makes clear that the process began with the geometrical construction of the pinnacle's ground plan within a square base. Next, a series of progressively smaller rotated squares should be inscribed within the original square. Further permutations of these figures, easily accomplished with the compass and straightedge, suffice to determine the complete ground plan of the pinnacle, including the collapsed 'footprint' of its vertical shaft. The elevation of the pinnacle was then determined by stacking up a series of modules based on the ground plan. This process of extrusion from the ground plan into the third dimension, which German authors call *Auszug*, or 'pulling out', was fundamental to the Gothic design method as a whole (see Shelby 1977: 76–79). Roriczer himself hints that something more general than pinnacle construction is at stake in his booklet. On its first page, he explains to his learned patron Wilhelm von Reichenau, the Bishop of Eichstätt, that his writings will 'explain the beginning of drawn-out stonework — how and in what measure it arises out of the fundamentals of geometry through manipulation of the dividers' (Shelby 1977: 82–83).[7] Significantly, too, he makes clear that he is describing traditional design methods rather than his own innovations, since he invokes the authority of the 'Junkers of Prague', the members of the Parler family who dominated architectural practice in central Europe from the middle of the fourteenth century. Roriczer may well have chosen to focus on pinnacles for basically pedagogical reasons, thinking that this simple example could clarify design principles of wide applicability, but the seeming narrowness of this topic surely diminished the impact of his writings. The tediously detailed quality of his text, which combines the worst qualities of a math textbook and a cookbook, also must have limited the appeal of his work.

Three decades after the publication of Roriczer's booklet, the noted Heidelberg court architect Lorenz Lechler tried to explain Gothic design more quickly and economically in his *Unterweisungen*, a more comprehensive compendium of architectural advice for his son Moritz (Coenen 1990: 15–25 and 146–152). Although Lechler's known architectural works, such as the sacrament house of S. Dionys in Esslingen, are formidable in their geometrical complexity, his writings present mostly short rules of thumb based on simple numerical ratios. He recommends, for example, that side aisle spans should be one half as great as the free span of the main central vessel, which he takes as his fundamental module. The thicknesses of the walls and piers, he suggests, should equal one tenth of this module. The capitals of the main vessel should sit either one module, or alternatively one and a half modules, above the floor. Lechler's short modular recipes are less tiresome to read than Roriczer's detailed geometrical instructions, but they ultimately prove frustrating to the modern researcher, since they fail to explain the origins of the complex dynamic forms that make German late-Gothic design so interesting. These examples, moreover, are unillustrated, at least in the three surviving manuscripts of the *Unterweisungen*. Lechler's manuscripts do, however, include several illustrations showing how combinations of geometrical and arithmetical subdivision could be used to generate the cross sections of window mullions. The small and large mullions are shown to have lengths of 5 and 7 units respectively, which at first sounds like a simple modular relationship. However, the smaller mullion is shown within a square circumscribed by a circle framed by a large square, which demonstrates that the 5:7 ratio is really just an approximation to the $1{:}\sqrt{2}$ proportion that emerges geometrically from the operation of square rotation, which is often called quadrature.[8] Gothic architects used a wide variety of similar constructions, often inscribing other regular polygons within circles to set the proportions of their building components.[9] They could also easily unfold the diagonals of a half-square to create the so-called Golden Ratio ϕ, which relates a whole harmonically to the sum of its parts.[10] Convenient numerical approximations to the resulting lengths might then be chosen to facilitate construction using fixed foot units or blocks of standardized sizes. Franklin Toker (1985) has aptly called this design method 'pseudo-modular', but Lechler provided no very clear exposition of the relationship between geometry and modularity in his work. The late medieval design handbooks, in fact, are fairly cavalier about theoretical niceties. Like many other medieval technical texts, in fact, they are essentially just compilations of recipes, rather than polished treatises with clear organization and argument structure.

In terms of scope and rhetorical sophistication, late medieval design booklets like Roriczer's and Lechler's were no match for the comprehensive architectural treatises that began to emerge contemporaneously in Renaissance Italy.[11] Vitruvius's impressively comprehensive and genuinely Roman *De architectura* had been known throughout the Middle Ages, but it received greater attention after its popularization by Poggio Bracciolini early in the fifteenth century (Kruft 1994: 38–39). Leon Battista Alberti wrote its most direct Renaissance successor, *De re aedificatoria*, in eloquent Ciceronian Latin that would appeal to well-educated humanist courtiers, in a way that the Gothic design booklets never could. Alberti's discussion of proportion, meanwhile, emphasized fixed whole-number ratios, rather than the more flexible relationships that could emerge in the geometrical dynamics of the Gothic tradition.[12] The proportions of Renaissance buildings, therefore, could be captured much more readily in simple graphics than those of Gothic buildings. This fact contributed to the success of illustrated Renaissance treatises, including most notably those produced by Serlio, Palladio, and Vignola, whose publications helped to spread Italianate architecture throughout Europe in the sixteenth century. From this perspective, the eclipse of the Gothic tradition can be understood in part as a consquence of its practitioners' inability to provide verbal and visual explanations for their methods as compelling and accessible as those provided by their Renaissance rivals.

Over the past five hundred years, therefore, the logic of the Gothic design process has been less well understood, and less celebrated, than that of classical architecture. While the module-based systems of classical design were actively taught to generations of students, most classically inclined writers followed Vasari's lead in dismissing Gothic architecture as lawless and disproportionate. Romantic writers who were more sympathetic to the Middle Ages, meanwhile, often saw the seeming freedom of the Gothic tradition as a virtue; thus they rarely devoted sustained attention to figuring out the logic of the Gothic design system. Although a fairly substantial literature on the topic had begun to emerge by the middle of the twentieth century, two complementary problems kept this work from enhancing the relative prestige of Gothic builders. First, rigorous historians like James Ackerman demonstrated that Gothic planning methods could be strikingly unsystematic and ad hoc. To add insult to injury, Ackerman's famous article on the chaotic progress of Milan Cathedral appeared in 1949, the same year that Rudolph Wittkower's *Architectural Principles in the Age of Humanism* argued for a close connection between the modularity of Renaissance design and the elegant harmonies of musical theory (Ackerman 1949; Wittkower 1949). The second and larger problem with research on Gothic proportion is that much of it appeared fanciful and unreliable, revealing more about the preconceptions of its authors than about the working methods of the Gothic designers themselves. In his magisterial 1960 review of writings on the Gothic period, therefore, Paul Frankl wrote in apparent frustration that 'the question of what is actually gained by such research becomes urgent. There can be no doubt that Gothic architects made use of triangulation and the like, but the excogitated networks made up of hundreds of lines to determine all points has not been proved and is probably undemonstrable and unlikely' (Frankl 1960: 722). The most devastating critique of this research tradition came from Konrad Hecht, whose work occupies a singular place in the historiography of Gothic proportion.

Hecht, writing in the years around 1970, aggressively challenged the authors who had tried to explain Gothic design in geometrical terms. Hecht argued that Gothic builders used a modular and numerical approach, rather than geometry, to define the proportions of their buildings. Hecht paid particular attention to the tower and spire in Freiburg im Breisgau, which had figured prominently in many earlier studies of Gothic proportion. Taking advantage of a recent survey, Hecht effectively demonstrated that most previously proposed geometrical theories about the spire's proportions were untenable. This fact, of course, does not mean that the Gothic builders of the tower did not use geometrical methods, but Hecht argued vociferously in this direction. To provide an alternative framework, Hecht attempted to show that module use could explain the proportions in the Freiburg tower, and in the elevation drawings for the tower of Ulm Minster.[13] Hecht's critique of poorly done geometrical scholarship was well motivated, but his modular schemes explain very little, since he gave no reason why their proportions should involve the modules he proposed. He was, in essence, just presenting numerical approximations to sets of proportions that could easily have been determined by geometrical means. His analyses thus amount to little more than quantified descriptions, which give no insight into the form-giving strategies used by medieval designers. When Hecht tried to achieve precision, moreover, he generally did so by transforming his subject buildings and drawings into nearly indigestible tables of numbers, thus obscuring the visual relationships that would have been paramount for a medieval builder or draftsman. Because Hecht's densely argued critical writings outwardly appear so rigorous, though, they continue to discourage research on Gothic architectural geometry even today. The impact of his writings has been particularly pronounced in the German-speaking world, where such work had formerly flourished.[14]

Towards a new understanding of Gothic geometry

Despite the widespread skepticism that Hecht's work radically exemplified, research into the geometrical bases of Gothic architectural design has a great deal to offer. And, while the field has not thrived in the past half century, enough good work has been done in recent decades to demonstrate the potential of such research (see, for example, Wu 2002a). Most importantly, perhaps, scholars including Stephen Murray have demonstrated convincingly that the overall proportions of Gothic buildings can often be explained by fairly simple sequences of dynamically unfolding geometrical operations. As Murray, Toker, and Peter Kidson have begun to show, this geometrical approach to design was compatible with modular approaches to construction and building layout (see Murray and Addiss 1990; Kidson 1993; and Toker 1985). As noted previously, in fact, modular dimensions were often chosen to approximate geometrically determined proportions, as Toker's term 'pseudo-modular' effectively suggests.

A variety of new technical and methodological approaches are now beginning to converge productively, in ways that are allowing decisive steps forward in the study of Gothic architectural geometry. First, truly accurate building surveys are beginning to become more widely available. Some nineteenth- and twentieth-century surveys were already quite precise, and current scholars still have good reason to conduct careful manual surveys; the Regensburg Cathedral survey project and the work of Matthew Cohen described in this volume provide good recent examples of such work.[15] But the field of building measurement is being rapidly transformed by the spread of photogrammetric and especially laser-based survey methods. As Andrew Tallon's essay in this volume demonstrates, such methods can dramatically increase the precision of building surveys, putting the field of geometrical research onto a new and strongly reinforced empirical foundation. A second and closely related development has been the spread of computer-aided design (CAD) systems, which permit scholars to draw exact geometrical figures, and to compare them to the forms seen in medieval buildings.[16] Together, these trends are rendering obsolete the concerns about imprecision and sloppiness that had formerly engendered much well-justified skepticism of geometrical research; this can be seen in the supplement to this article (see http://journal.eahn.org/hosted/chevet-notre-dame-geometrical-analysis), concerning the plan of Notre-Dame in Paris. Even with the world's most precise building surveys, though, some ambiguity about the intentions of the designers remains, because errors and changes may have been introduced in the course of the construction process, and because it is not always easy to tell from the fabric of even a well-constructed building which elements had conceptual priority for the designers. For these reasons, the study of surviving medieval architectural drawings can be a helpful adjunct to the study of the monuments.[17] Drawings are the documents, after all, that were produced by the designers themselves. Their proportions thus tend to reflect the designer's intentions more directly than the buildings do.[18] Drawings, moreover, include blind lines, compass prick marks, and other traces of the draftsman's labor, which can help to reveal the logic of the design's conception. The scribed lines often marking pier and buttress centerlines, for example, clearly attest to the importance of these axes in the layout of the drawings. The vast majority of the visual information in a drawing, though, appears in the inked lines describing the architectural forms themselves, which have rarely received the careful geometrical scrutiny that they deserve.

It is crucial to recognize, in this context, that dynamic geometry was not simply a means that Gothic designers used to establish the overall proportions of their buildings; rather, it was a comprehensive form-giving strategy that determined the shapes of individual building components as well as the relationships between large- and small-scale forms. The geometrical steps of the Gothic design process, of course, did not take place in a vacuum. Tradition, functional requirements, and educated guesses about structural stability, all would have informed the design process, establishing the basic outlines of the architectural scheme in ways that geometry by itself never could. Most Gothic designers, therefore, probably had at least a rough idea in mind even before sitting down at the drafting table. Geometrical experimentation with the compass and rule then served to sharpen the focus, by generating specific trial lines that could be accepted or rejected depending on their usefulness in the overall scheme. In a sense, therefore, a Gothic design can be seen as an architectural topiary, in which geometry provides the quasi-random growth factor, while artistic judgment guides the pruning process. This dialog between growth and pruning helps to explain the organic quality characteristic of Gothic design.

With this perspective in mind, it becomes possible to achieve a geometrically informed understanding of Gothic proportion far more plausible, and far more satisfying, than the strictly modular accounts provided by antigeometrical skeptics like Hecht. The investigative method employed in the following case studies, therefore, closely emulates the Gothic design process just described. Here, once again, basic geometrical operations have been used to generate trial lines. In this context, though, the importance of a line can be judged by how well it matches lines already determined by the medieval designers, rather than by how well it matches a vague phantom in the mind's eye. This distinction, of course, makes the investigative process less open-ended than the original design process, but the resonance between the two has great methodological importance. In order to generate plausible hypotheses for testing, the researcher has to empathize with the original designer, imagining how a given design can be brought forth step by step on an initially blank sheet.

The following case studies show how the use of CAD systems permits both the fruitful harnessing of this creative empathy, and the rigorous testing of geometrical hypotheses. All of the associated graphics were created using the Vectorworks CAD system, in a three-stage process. First, source images of the drawings or buildings in question were scanned and imported into the system. Second, their relative proportions were carefully checked against published dimensions and measurements made in the field, and corrected where necessary; these adjustments were generally quite small, thanks to the quality of the source images.[19] Finally, the CAD system was used to draw trial lines and polygons on top of the source images. The geometries of these added lines are perfect, in the sense that the squares are square, the circles circular, the verticals vertical, and so forth. These figures, in other words, have never been adjusted or 'fudged' to match the source images. The computer, moreover, treats these figures as assemblages of perfectly thin lines, so that the user never has to worry about finite line width introducing imprecision into the geometries.[20] The goal in creating all of these figures was to find coherent sequences of geometrical operations that would cumulatively built up the outlines of the medieval forms. In cases where original design drawings survive, the presence of compass prick marks and blind lines provided valuable evidence about the constructions actually used by the medieval draftsmen, as noted previously. The combination of CAD use and careful on-site examination of drawings, therefore, minimizes the problems of imprecision and ambiguity that had troubled critics of earlier geometrical research. This method, in fact, allows modern researchers to test geometrical hypotheses with unprecedented rigor.

The graphics in the rest of this essay, and in the larger study from which they have been drawn, are meant to illustrate the geometrical logic of the designs in question.[21] They thus explicitly show geometrical figures to make visible operations that the original draftsmen likely used in creating their design drawings. The draftsmen themselves, however, would not have had to draw complete figures like these in order to establish the layout of their compositions. A designer wishing to establish points outside an already constructed square, for example, might have used his compasses to unfold the diagonals of the square to its baseline, but he would have had no need to actually draw in the arcs describing the path of the compass. Indeed, he would have had good reason not to, since such visible arcs would have appeared intrusive and distracting in the final drawing.[22] So, while Gothic drawings and buildings have a strongly geometrical character, the logic of their designs becomes apparent only when extra lines and figures are superimposed over them, as the following case studies will demonstrate.

Confronting the Hechtian legacy at Ulm and Freiburg

It makes sense to begin this geometrical discussion with consideration of the Ulm and Freiburg tower projects, both because of the prominent roles they played in Hecht's discussion, and because the pinnacle-like format of these towers facilitates comparison with Roricizer's pinnacle design booklet. Since Hecht recognized that the analysis of original drawings can provide an even more intimate perspective on the medieval design process than the analysis of buildings, he dedicated the culminating chapter of his book on Gothic proportions to the great elevation drawings associated with the Ulm Minster workshop. This decision made good sense, not only because these drawings are among the most spectacular of medieval 'blueprints' but also because they can be fruitfully compared with the structure of the present tower, whose construction they guided. Hecht's analysis of the Ulm elevation drawings must be criticized as perverse and unhistorical, however, not only because he chose to atomize these masterpieces of Gothic draftsmanship into tables of numbers, but also because he ignored much of what medieval sources reveal about Gothic design. Since Roriczer's first step in designing a pinnacle was to establish its ground plan, and since ground plans also have priority over elevations in the booklets written by his near-contemporary Lorenz Lechler, it is odd that Hecht chose not to analyze the ground plans associated with the Ulm tower project.

Two closely related plan drawings survive to document early planning on the Ulm tower. One drawing, now preserved in London, shows the tower mostly at ground level, while another, which remains in Ulm, shows mostly the transition to the octagonal story (**figs. 2b** and **2a**).

Until recently, both were generally dated to the 1390s and associated with the career of the first designer involved with the project, Ulrich von Ensingen.[23] In their recent catalog of drawings from the Ulm region, Hans Böker and his team have plausibly proposed later datings, attributing the plans to two of Ulrich's followers, Hans Kun and Matthäus Ensinger, but both drawings clearly reflect the geometrical givens established by Ulrich von Ensingen in his design for the tower base and its buttresses.[24] Both drawings fit neatly into the same geometrical framework, which is shown in **Figures 2c** and **2d**. Within the basic square footprint of the tower, the walls and buttresses are one fourth as wide as the open space between them, so that the salience of the buttresses beyond their centerlines equals one tenth of the interval between those centerlines. This simple modular relationship, shown by the small dotted arcs at the top of **Figure 2c**, echoes the recommendations for wall thickness published in Lechler's booklet.[25]

Within this simple modular armature, though, Ulrich von Ensingen soon constructed complex geometrical figures whose subtleties would go on to influence all later contributors to the tower project. Most obviously, he constructed octagons within the square framework of the tower base, establishing the basic symmetry pattern for the tower and spire superstructure. The smallest octagon visible in **Figure 2a** stands slightly but measurably inboard of the buttress edges, corresponding to the dotted octagon shown below in **Figure 2c**, rather than to the solid lines framing the buttresses. As the labels at left indicate, their distances from the tower center are 0.765 and 0.800 times as great, respectively, as the distance to the buttress axes, which can be called one unit for convenience. The large dotted circle in **Figure 2c** illustrates the relationship between these geometries. The radius of the circle is established by the point where the rays aiming for the octagon corners intersect the centerlines of the main buttresses; these points are indicated by the larger black dots in the figure. The large circle thus defined then sweeps through the principal diagonals of the tower plan, creating the intersection points shown by the smaller black dots in the figure. These points define the corners of the dotted square in **Figure 2c**, which frames the dotted octagon corresponding to the inner octagon shown in **Figure 2a**. This octagon stands inset from the buttresses, since the 0.765 unit span determined by this unfolding geometrical construction differs from the 0.800 unit span given by the simple modular frame of the buttress outlines. The proportional relationship between the tower octagon and the buttresses, in other words, can only be understood by considering the interaction of modular and geometrical design strategies.

Figure 2d shows how a similar construction explains one crucial subtlety of Ulrich von Ensingen's tower design; namely, the way the tower buttress axes pinch inward above ground level. The white dots in the figure indicate the points where a large circle inscribed within the overall tower footprint intersects the principal diagonals and the rays to the octagon corners. Lines projected forward from these intersection points define the edges of the buttresses in the second tower story. The inner and outer edges are 0.849 and 1.109 units from the building centerline, respectively, as the labels along the bottom of the figure show. The centerlines, shown in bold in the figure, thus stand 0.979 units from the tower centerline. So, as the heavy lines within the salient buttresses indicate, their centerlines are indeed slightly inboard of the original dotted buttress axes defined at ground level. The inward stepping of the buttresses that results can be seen not only in the Ulm ground plans, and in the tower itself, but also in the elevation drawings that helped to guide its construction.

The medieval elevation drawing most closely related to the final form of the Ulm tower is the so-called Ulm Riss C, created by Matthäus Böblinger around 1477 (**Fig. 3**).

Böblinger necessarily took as his point of departure the proportions established by Ulrich von Ensingen at ground level, and the structure of the tower base erected in the first three quarters of the fifteenth century, but he modified the design by introducing a taller belfry story and a simpler overall silhouette than his predecessors had foreseen. Böblinger himself was unable to finish the tower because of structural problems that arose in the 1490s, but his Riss C eventually went on to inform the nineteenth-century campaigns that made the spire the world's tallest masonry structure upon its completion in 1890. Discussion of all the drawing's intricacies would take more space than this short essay permits, but several basic points deserve emphasis.[26] First, the geometrical armature shown in **Figure 3** explains the forms of the drawing with great precision. This is evident, for example, in the upper zone, where the spire is drawn within

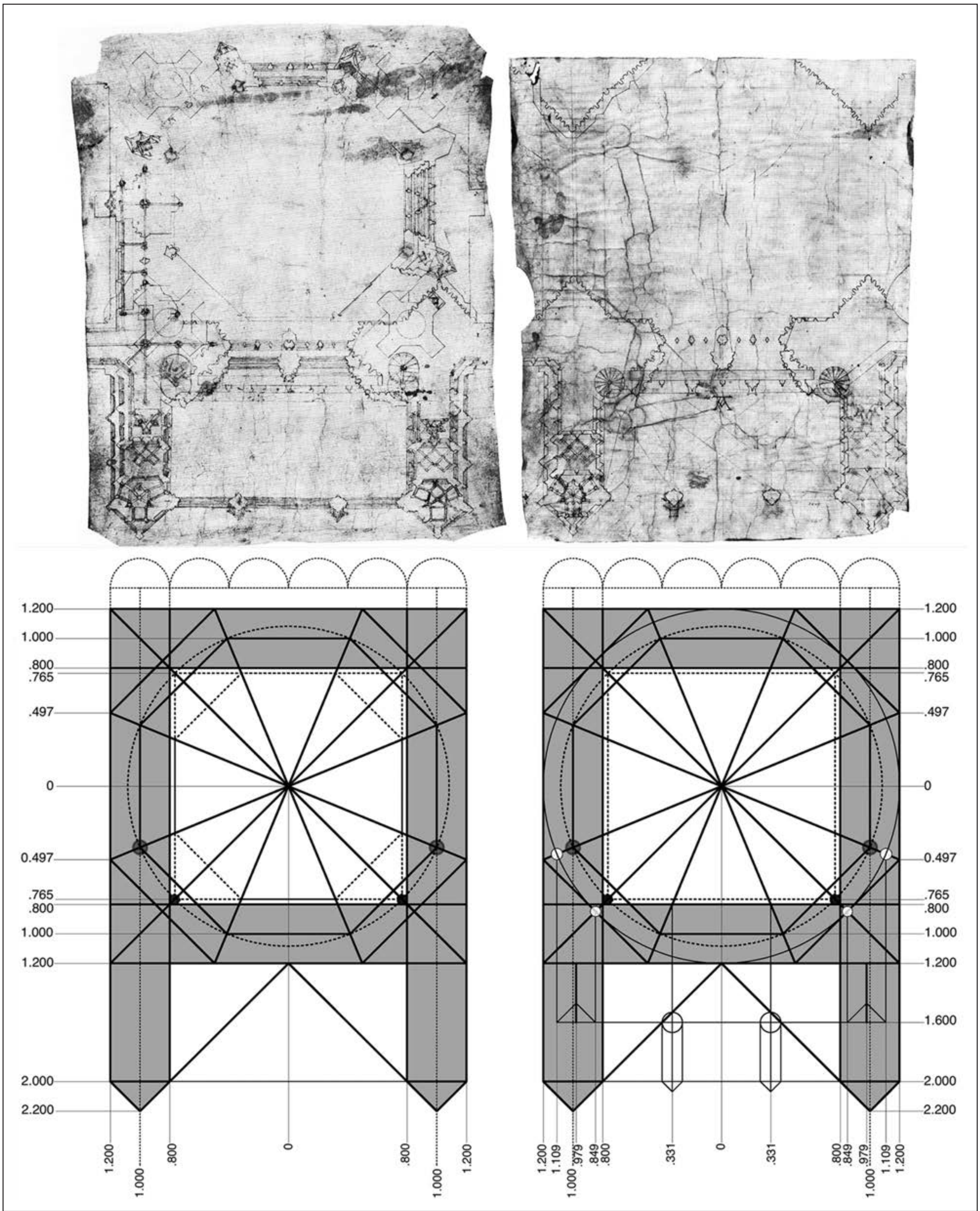

Fig. 2: a) (top left) Plan drawing of Ulm Minster's west tower, from Ulm, Archiv des Münsterbauamtes. Photo: Friedrich (1962); **b)** (top right) Plan drawing of Ulm Minster's west tower, from London, Victoria and Albert Museum. Photo: Friedrich (1962); **c)** (bottom left) Basic geometrical scheme for plan of Ulm Minster's west tower. Image: author; **d)** (bottom right) Elaborated geometrical scheme for plan of Ulm Minster's west tower. Image: author.

a stack of three equally sized squares, with the successive horizontals in this stack locating the crockets flanking the spire cone. More subtly, the overall geometrical armature appears to have also governed details such as the location of stringcourses and tracery panels in the buttress articulation. These relationships, and similar ones seen in many other drawings, demonstrate that the placement of Gothic architectural ornament often reflected the underlying geometrical logic of the whole design. So, while Gothic ornament can appear strangely flexible and

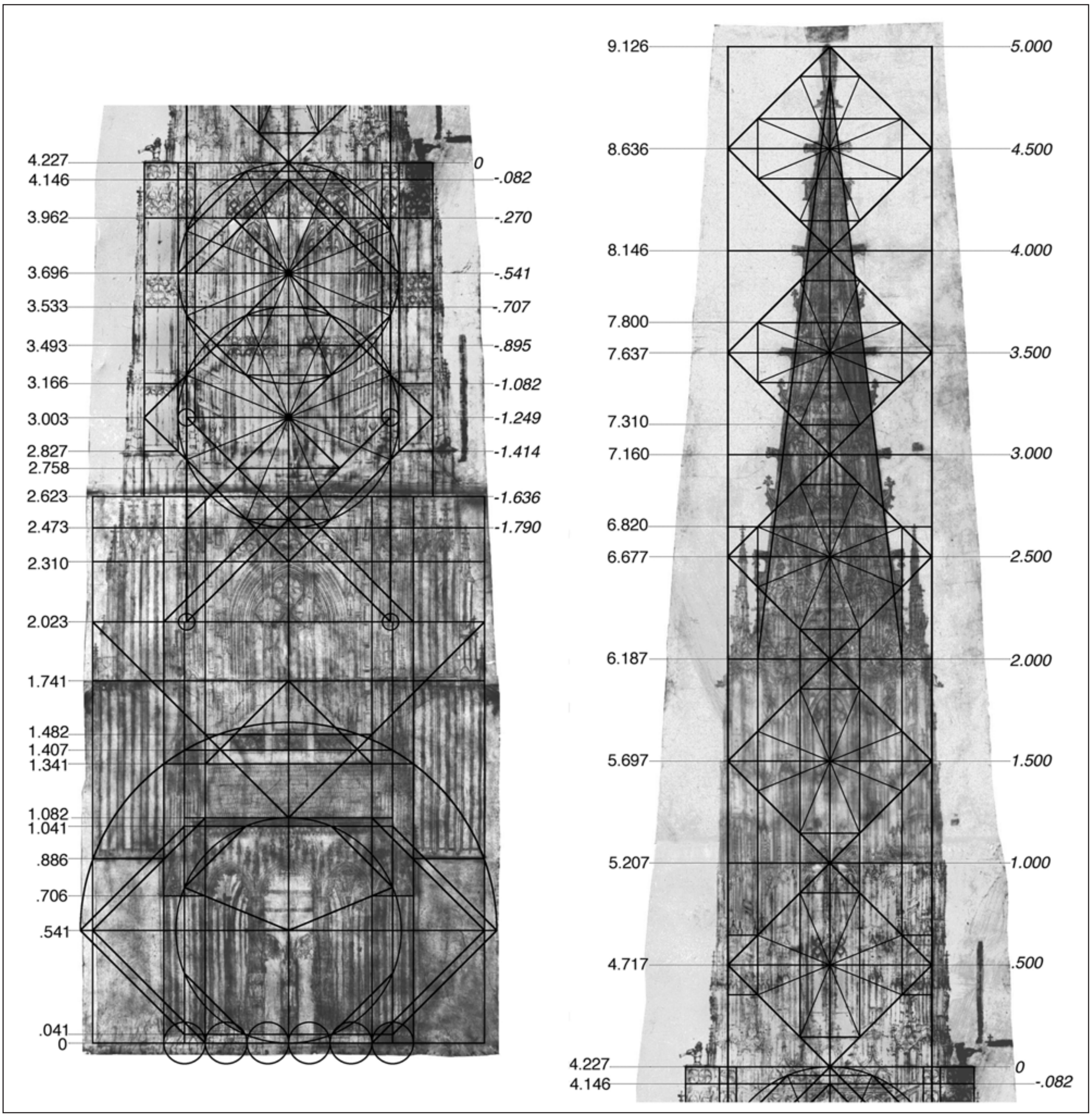

Fig. 3: Elevation drawing of Ulm Minster's west tower (Ulm Riss C), by Matthäus Böblinger, c. 1477, with geometrical overlay by the author. Photo: Stefan Roller and Ulm, Stadtmuseum.

capricious, especially when seen from a classical perspective, its deployment was anything but random. Further evidence for the importance of the geometrical armature in Böblinger's Riss C comes from the arrangement of its constituent parchment pieces. Thus, for example, the parchments in the top part of the drawing were left wide enough to include the two major vertical axes framing the cage of diagonal lines in the upper spire zone. These lines correspond precisely to the pinched buttress axes introduced by Ulrich von Ensingen in his designs for the tower plan, already described in **Figure 2d**. These are also the lines used to define the square modules stacked in the spire zone.[27]

It thus becomes clear that Ulm Riss C incorporates the same mixture of geometrical and modular design principles described in Roriczer's booklet on pinnacle composition, but in more complex and convoluted form. In both cases, the forms established in the ground plan go on to influence the elevation through processes of extrapolation and stacking. These findings thus help to demonstrate how the simple exercises described in late medieval texts related to actual building projects, including even the most ambitious tower-building projects of the era.

Since the Ulm tower was in many ways just an updated and enlarged version of the tower at Freiburg im Breisgau, it would be natural to suspect that some of the same design strategies were at work there. Geometrical analysis demonstrates that this was indeed the case, even though the Freiburg project began already in the late thirteenth century. The base of the Freiburg tower, which is quite spartan in appearance, was probably designed around 1270. The lacy upper tower and openwork spire, though, clearly reflects a different vision, suggesting strongly that it was designed only around 1300, with construction of the spire lasting through the first quarter of the fourteenth century.[28] The Freiburg tower is not as well documented in original design drawings as the later Ulm tower. Seven medieval drawings depict variants of the Freiburg design, but since none of them relates very precisely to either component of the structure, Hecht left them entirely out of his account. In recent years there has been a growing recognition that these drawings, even if they postdate the spire, may record valuable information about the logic of its conception. The drawing that holds inventory number 16.869 in the spectacular collection of the Viennese Academy of Fine Arts, in particular, records a scheme likely connected with an intermediate phase of the Freiburg, project, conceived between the completion of the tower base and the design of the far more complex tower superstructure.[29] Geometrical analysis of this drawing and of the two main components of the tower supports this conclusion, demonstrating both continuity and development in the Gothic design tradition.

The lowest section of the Freiburg tower is not only the oldest part of the structure, but also the simplest, which makes analysis of its proportions comparatively straightforward. The tower base is a plainly articulated masonry box, with two buttresses emerging from each face. The corners of the box are just visible between adjacent buttresses, forming a salient masonry flange in the space between them. Some of the proportional relationships between these components are quite obvious. The span across the outer faces of the lateral buttresses, 12.39 meters, is almost exactly twice the 6.19-meter span between the axes of the forward-facing buttress. Hecht believed that these dimensions were to be understood at 80 feet and 40 feet respectively, with the size of his postulated foot units being based on a convoluted and ultimately implausible statistical argument. The span between the box corners, 15.71 meters, he described as 50 feet 6 inches, without suggesting any rationale for why the tower designer would have chosen this dimension (Hecht 1979: 344).

As the lower portion of **Figure 4a** indicates, a straightforward geometrical construction involving the proportions of the square and the equilateral triangle suffices to determine the width to the corner flange of the box. A line with a 30-degree slope departing from the base of the trumeau intersects the outer buttress face at the 1.15-unit height, where a unit is defined once again as the space between the building centerline and the axis of the forward-facing buttress. A shaded triangle fills the space between this line and another, with a slope of 45 degrees, that rises from the trumeau base to intersect that buttress axis at height 1.00, before bouncing down to meet the outer buttress face at its base. The right-hand corner of the shaded triangle, which is the intersection point between the falling 45-degree line and the rising 30-degree line, falls 1.268 units to the right of the building centerline. This simple construction thus defines the width of the basic box even more precisely than Hecht's ad hoc numerical description.[30] With these fundamental dimensions in hand, many other elements in the tower base can be located. The horizontal moldings at height 2.15 and 3.15, for example, are found by stacking 1.00 unit boxes on the already established baseline at height 1.15. The span to the outer buttress face after its first setback is 1.793 units, which is exactly $\sqrt{2}$ larger than 1.268; this can be seen in the large arc at the top of the figure, which also sets the chapel height up to level 4.95.

It is interesting and significant that many of these same elements recur, in somewhat altered form, in the elevation drawing number 16.869 (**Fig. 4b**), which may well record the oldest surviving design for Freiburg's openwork spire.[31] As **Figure 4b** shows, the tower base depicted in the drawing differs slightly in its proportions from the built structure, and its portal gable is more sharply pitched, with larger and more florid crockets. These elaborated details suggest that the drawing postdates the construction of the tower base. Since the main purpose of the drawing was probably to present a design for the tower superstructure and spire, its creator does not appear to have been concerned about creating an absolutely precise depiction of the tower base.

As in the present tower base, though, the intersection of 30- and 45-degree lines seems to have been used to set the 1.268 span to the corner flange in the drawing. This dimension was then added twice along the vertical axis to locate the horizontal moldings at heights 2.42 and 3.68, which thus rise measurably higher than their analogs on the real tower, where the stacked elements are only one unit high. In the drawing, moreover, the large arc at the top of the figure sets the height of the chapel including its terminal balustrade; the same strategy was also used in Böblinger's Ulm Riss C.[32]

Above this first balustrade drawing 16.869 shows a large belfry zone topped by a second balustrade, which is not seen in the present Freiburg design (**Fig. 5b**). This discrepancy has raised questions about whether the scheme in the drawing predates or postdates the actual tower superstructure. The coherent and almost facile geometry of the 16.869 design supports the former reading.[33] In the drawing, the buttress axes continue uninterrupted past the first balustrade, and the belfry zone appears to have a simple square plan. The belfry is also a perfect square in elevation; its height and its width are both twice the 1.268 dimension established in the tower base. The belfry thus rises between heights 5.48 and 9.07, measured between the tops of the two balustrades.

Above the second balustrade, the upper tower and spire in the drawing fit precisely into a stack of four square modules, each 1.268 units per side. The corners of an octagon inscribed within the lowest of the four locates the corner flanges of the octagonally symmetrical story just below the spire base. As in the case of Ulm Riss C, the parchment is squared off at the top, so as to encompass the full rectangular armature. And, as in the Ulm drawing, the spire fits into a stack of three boxes. While the crockets of Ulm Plan C counted out this rhythm, though, that role falls in the Freiburg drawing to the tracery roundels of

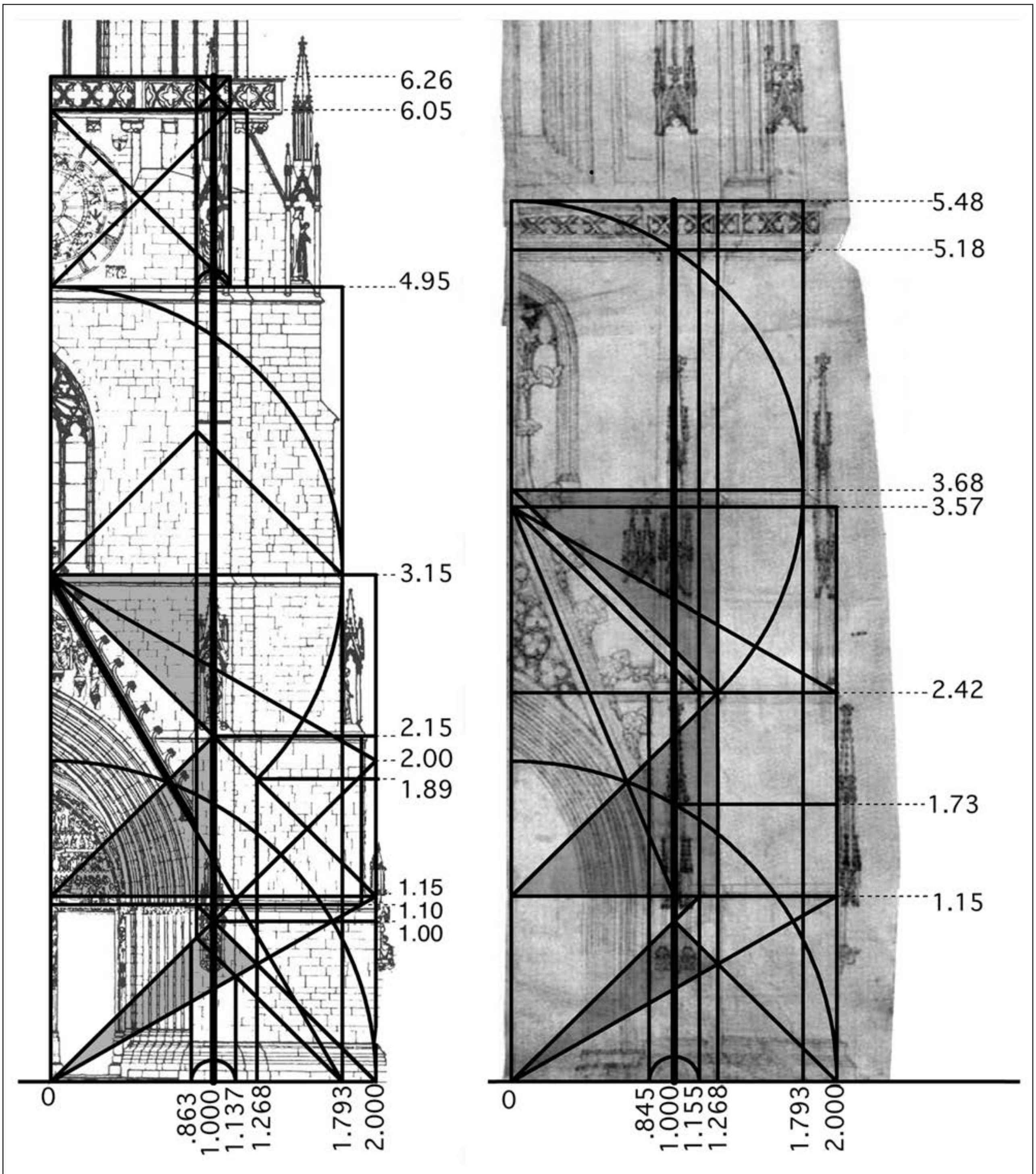

Fig. 4: a) (left) Elevation of Freiburg tower base, with geometrical overlay by the author. Photo: Freiburger Münsterbauverein e.V.; **b)** (right) Elevation drawing 16.869, lower portion, with geometrical overlay by the author. Photo: Vienna, Kupferstichkabinett der Akademie der bildenden Künste.

the spire, two of which are centered at the box-bounding heights 12.87, 14.14 and 15.41. The tip of the spire finial is at height 19.21 units; if the drawing were scaled so that the distance between its buttress axes measured the same 6.19 meters seen in the present Freiburg spire base, this would work out to an overall height of 118.91 meters.

The present Freiburg spire is slightly shorter than the one depicted in 16.869, but its geometry is far more sophisticated, suggesting that the present design was developed later. And, while the geometry of the drawing is quite consistent from ground level to spire tip, the complex format of the actual upper tower and spire differs markedly from the simple boxy format of the tower base. As **Figure 5a** shows, the buttresses of the tower base were abruptly terminated in a short transitional zone capped by a single balustrade that runs between heights 6.05 and 6.26. In plan, this balustrade describes a complex twelve-pointed star. Its format can best be understood as the result of the dynamically unfolding process illustrated in **Figures 6a–f**.[34]

As **Figure 6a** shows, the basic frame of the figure is a square circumscribed about an octagon, a circle, and a smaller square, whose corners coincide with the corner pinnacles flanking the octagonal tower core; these pinnacles lie 1.000 units out from the building centerline, so that they align with the axes of the buttresses in the tower base below. However, while the designer of the tower base used combinations of square and equilateral triangular geometries in elevation, the designer of the superstructure combined these figures in the plan. So, as **Figure 6b** shows, the basic star shape within the frame can be found by drawing wedges 30 degrees wide within the 45-degree wedges created by the octagonal geometry of the overall plan. Then, as **Figure 6c** shows, equilateral triangles can be inserted into the four corner wedges, forming the basic twelve-pointed figure. Further elaborations in **Figures 6d** and **6e** produce the final form shown in **Figure 6f**, which agrees superbly well the the plan of the tower as recorded in survey drawings. The basic dimensions established in the plan, moreover, can be stacked to give the crucial points in the elevation, which are shown in **Figure 5a**.[35] Here once again, the dynamics of geometry provide an explanation for the proportions of the structure far more compelling, and far more historically plausible, than Hecht's ad hoc modular schemes. The Freiburg and Ulm spire designs both involve design strategies very similar to those seen in Roriczer's pinnacle booklet.

Polygons and 'irrational' proportions in Gothic church elevations

Geometrical design strategies were used throughout the Gothic era to set the proportions not only of pinnacle-shaped spires, but also of church cross sections. In the literature on Gothic design, such sections are often described as being designed either *ad quadratum* or *ad triangulum*, i.e. to the proportions of a square or to those of a triangle. This simple binary, of course, hardly suffices to describe the full palette of options employed by Gothic designers. As Ackerman showed decades ago in the case of Milan, even the term *ad triangulum* could have a variety of meanings, depending on whether they involved equilateral triangles or other types, and depending on how these geometric figures were applied in relation to the elevation. Ackerman's article on Milan also placed great emphasis on the efforts of the mathematician Stornaloco to find a modular approximation for the proportions that result from the construction of an equilateral triangle, which are called 'irrational' in the mathematical sense because they cannot be expressed as a ratio of whole numbers (Ackerman 1949: esp. 90–96).[36] To gain a complementary perspective on this issue, the following paragraphs present case studies of several buildings whose elevations appear to have been governed by great octagons: the cathedrals of Prague, Clermont-Ferrand, and Reims, and the Cistercian church of Altenberg. Octagons, of course, can be neatly inscribed within squares, but it would be too simple to describe any of these buildings as being designed *ad quadratum*, since their proportions evidently depend on the 'irrational' relationships deriving from the geometries of the octagons.

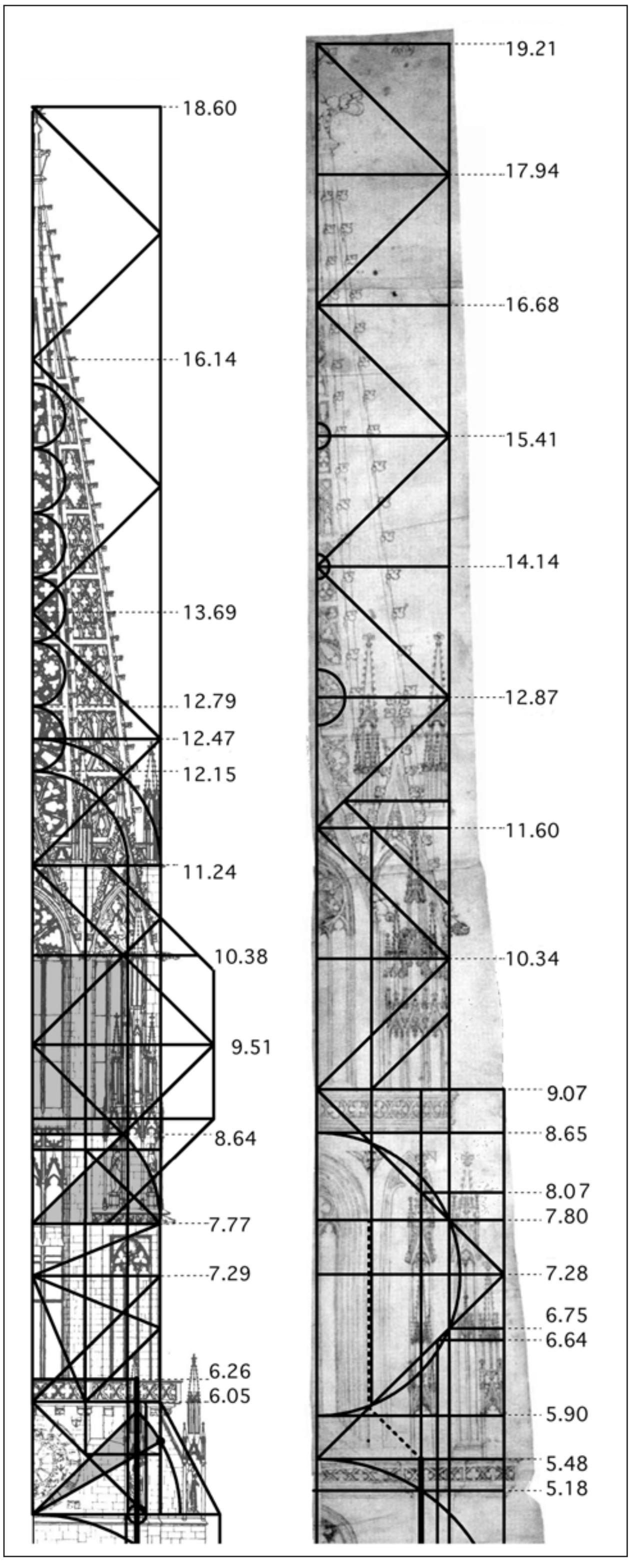

Fig. 5: a) (left) Elevation of Freiburg tower superstructure, with geometrical overlay by the author. Photo: Freiburger Münsterbauverein e.V.; **b)** (right) Elevation drawing 16.869, upper portion, with geometrical overlay by the author. Photo: Vienna, Kupferstichkabinett der Akademie der bildenden Künste.

The planning for Prague Cathedral deserves particularly close attention in this context, for several reasons: first, because much of the building was designed by Peter Parler, the first and most influential of the 'Junkers of

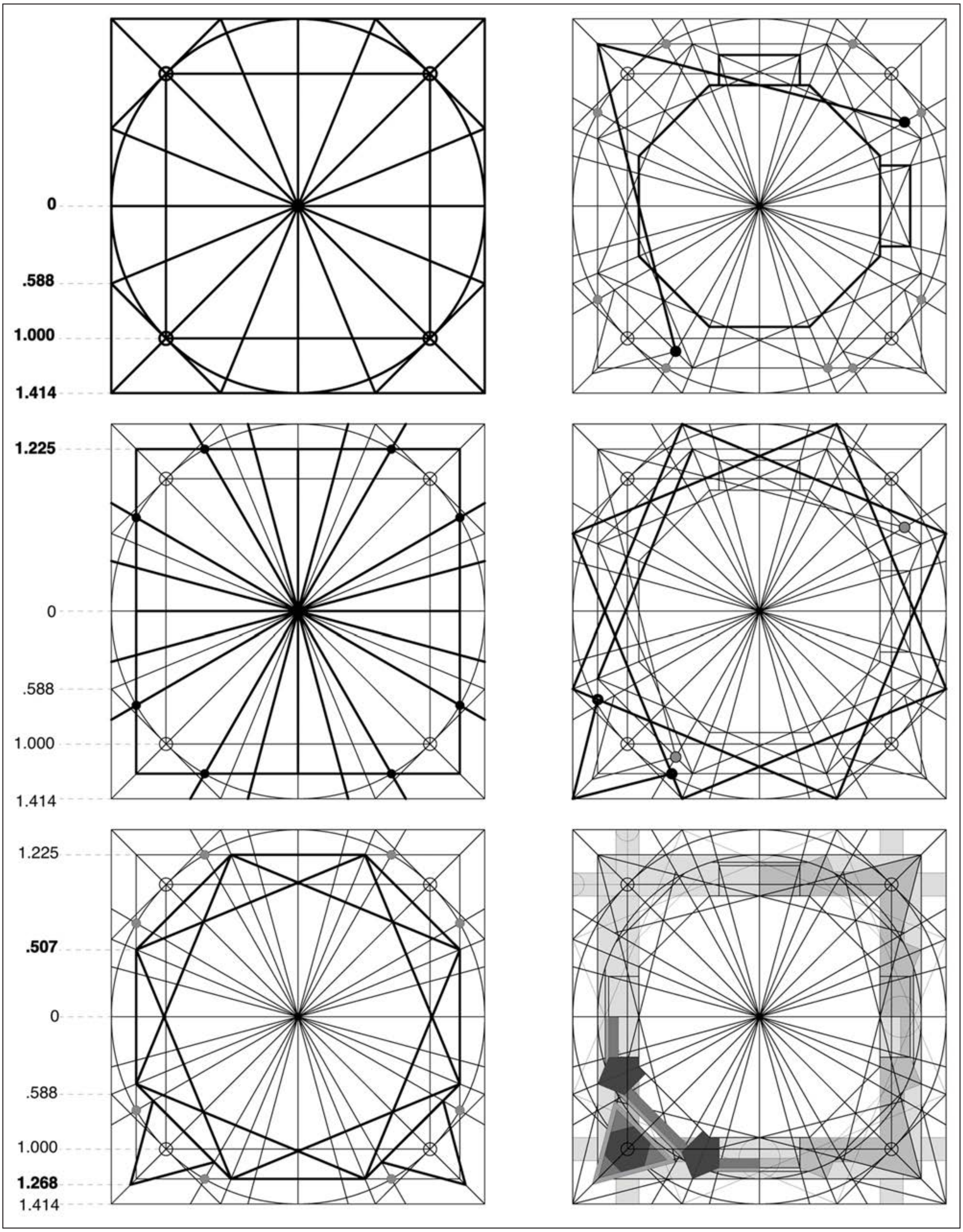

Fig. 6: Succesive stages in geometrical development of Freiburg upper tower cross section. Original graphics by the author. **a)** (top left); **b)** (center left); **c)** (bottom left); **d)** (top right); **e)** (center right); **f)** (bottom right).

Prague' cited by Roriczer as the authoritative practitioners of his Gothic tradition; second, because an original drawing survives to document the planning of the cathedral's section; and third, because analysis of this drawing helps to shed light on the relationship between Peter Parler and his French predecessor Matthias of Arras, who began construction of the cathedral in 1344. Comparison of the Prague section with those of Clermont-Ferrand, Reims, and Altenberg will show that the octagon-based planning strategy seen in the Prague drawing was already being

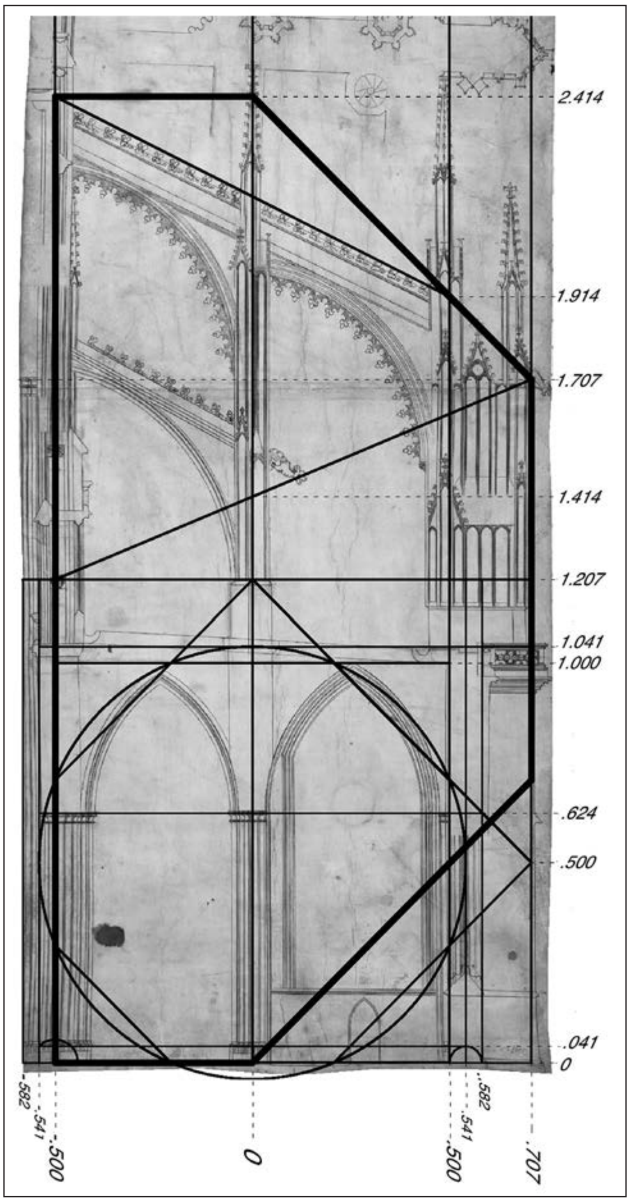

Fig. 7: Drawing 16.821 showing section of Prague Cathedral, with geometrical overlay by the author. Photo: Vienna, Kupferstichkabinett der Akademie der bildenden Künste.

used in France and Germany by the middle of the thirteenth century.

Drawing 16.821 in the Vienna Academy collection shows the cross section of Prague Cathedral's choir aisles and the flying buttressses that soar over them to brace the choir wall (see Böker 2005: 74–78; Bork 2011b: 207–212). The detailing of the buttresses is somewhat simpler than in the actual structure, suggesting that the drawing may have been produced under Peter Parler's direction fairly early in the design process. In this drawing, the overall proportions are set by the right half of a great octagon, whose height equals the span from the floor to the top of the upper flying buttresses (**Fig. 7**).

The center of the octagon coincides with a small mask that gazes out from the middle of the triforium. The ray from the center of the octagon to its upper right corner passes through the two gargoyles on the flying buttresses, which are thus used as geometrical markers. The height of this upper right corner coincides with the height of the capitals in the main elevation; this height can be called 1.707, where 1.000 is equal to the combined width of the two equally sized aisles. The aisles also rise to height 1.000, so that they fit into a square. When an octagon is inscribed within this square, and a rotated square placed around the octagon, its right-hand tip falls on the outer face of the lateral buttress. The midline of the outer wall aligns with a circle circumscribed about this octagon.

The geometrical principles governing the drawing were adopted quite faithfully in the actual choir structure, as **Figure 8** shows. The buttress articulation in the real building is more complicated, as noted above, and the intermediate buttress pinnacle now terminates a bit lower than in the drawing, but the proportions of the main elements are effectively identical. Importantly, too, this graphic shows that the central vessel of the Prague choir has proportions determined quite precisely by a single great governing octagon. These proportions occur for two reasons: first, because the geometry of the drawing uses a great half-octagon to relate the elevation of the main vessel to the width of the aisles; and second, because the central

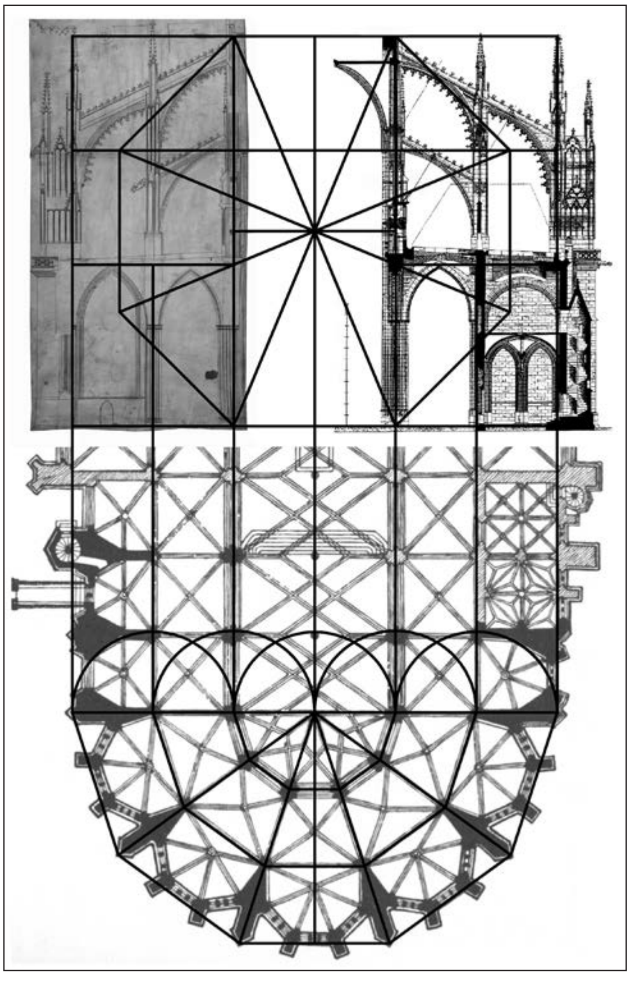

Fig. 8: Comparison of drawing 16.821 with Prague Cathedral's present section and ground plan after Podlaha and Hilbert, *Metropolitní chrám sv. Vita*, Fig. 68, and Burian, *Der Vietsdom auf den Prager Burg*, p. xix, respectively, with geometrical overlays by the author.

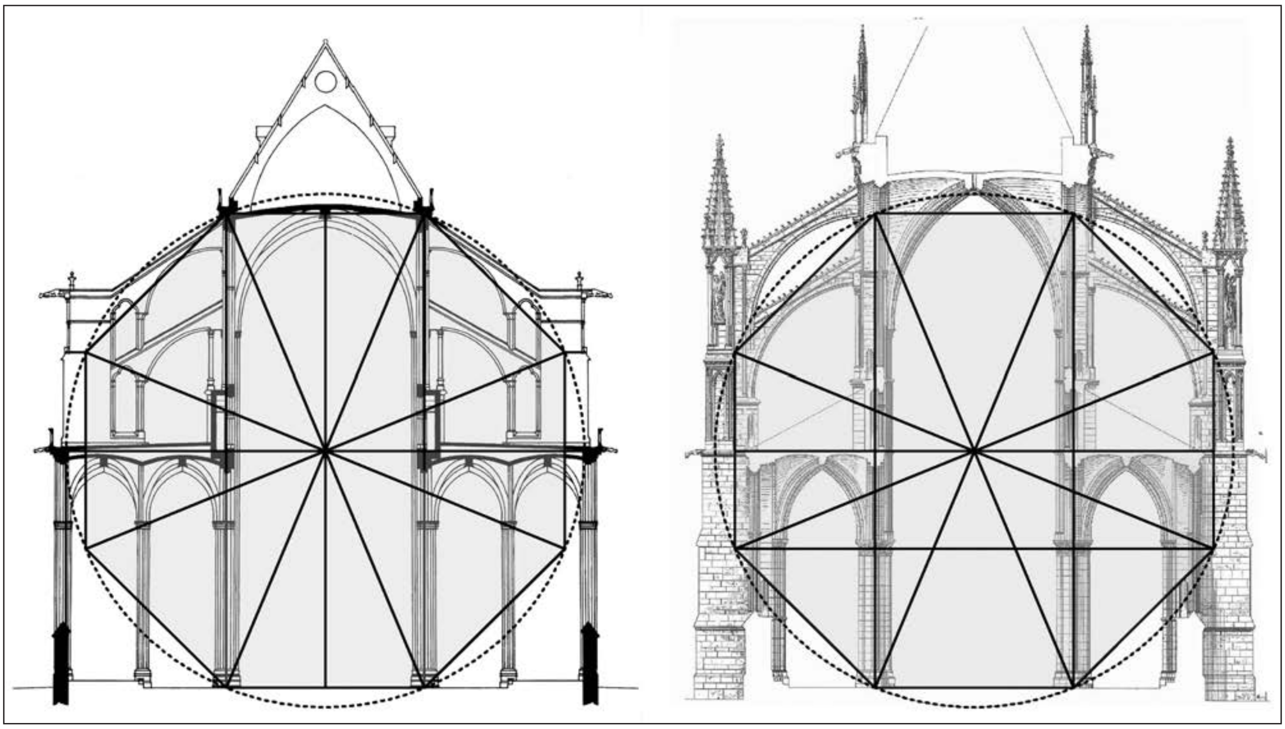

Fig. 9: a) (left) Clermont-Ferrand Cathedral, choir section, drawn by D. Fiegenschue after Henri du Ranquet (source: Davis, The Choir of the Cathedral of Clermont-Ferrand: Fig. 6), with geometrical overlay by the author; **b)** (right) Reims Cathedral, nave cross section after Dehio and von Bezold (1901), with geometrical overlay by the author.

vessel is exactly twice as wide as the aisles, as the ground plan in the bottom of the graphic shows. These results together mean that the half-octagon seen in **Figure 7** can slide over into the main vessel, where symmetry about the building axis then produces the full octagonal scheme seen in **Figure 8**.

Geometrical analysis suggests that Peter Parler owed more than has usually been imagined to his French predecessor Matthias of Arras. Matthias had designed the radiating chapels and the beginnings of the lower story in the straight bays of the choir; these are shown in dark grey in the ground plan, while the later portions completed under Parler's direction are shown in light grey. It was Matthias, therefore, who established the 2:1 relationship between the width of the main vessel and the aisles. And it was Matthias who began to define the elevation by establishing the height of the aisle and chapel vaults. But evidence from Prague cannot, by itself, say what Matthias intended for the upper stories. It is significant, in this connection, that precisely the same octagon-based geometry seen in Parler's drawing and in the present Prague choir also governs the proportions of the cathedral at Clermont-Ferrand, as **Figure 9a** shows.[37] Since Matthias worked in southern France before coming to Prague, he surely would have known Clermont Cathedral, which was begun in 1248. Matthias probably had the Clermont scheme in mind when he began the Prague project, for which he likely produced elevation drawings.

The octagon-based proportioning scheme seen at Prague and Clermont was already being used early in the thirteenth century to establish the elevation of Reims Cathedral, as **Figure 9b** shows. As at Clermont, the aisles teminate at the equator of a great octagon whose lower facet corresponds to the floor of the main vessel, measured between the arcade axes. In both cases, therefore, the proportions of the main vessel are 'irrational' in the mathematical sense, although the designs are geometrically quite lucid. At Reims, the corners of the great octagon establish the baselines of the capitals in the aisles, and the midlines of the capitals in the arcades. At Reims, the steeply pitched main vaults surpass the height of the great octagon's upper facet, which might at first seem to represent either a breakdown in architectural order, or a problem with the geometrical analysis. In fact, though, scrutiny of the vaults springers early in the twentieth century convinced Henri Deneux that the vaults were originally planned to be about 1.70m lower than they are today, which would place their keystones on the top facet of the octagon.[38] As **Figure 9b** shows, moreover, the current transverse arches now rise to meet the circle circumscribed around the octagon, demonstrating that even the vault revision took account of the building's overall geometrical order.

Since Reims Cathedral was greatly admired already in the thirteenth century, as many drawings by Villard de Honnecourt attest, it is not surprising that ideas from Reims soon began to influence the design of buildings not only in southern France, as at Clermont, but also in the German-speaking world. The octagon-based elevation scheme of Reims was copied, for example, at the Liebfrauenkirche in Trier, begun most likely around 1227, and at the Cistercian church of Altenberg, begun in 1259.[39] These projects demonstrate that the geometrical planning strategies seen at Reims and Clermont had begun to enter the Germanic world a century before Matthias of Arras began his work at Prague. At Altenberg, the proportions

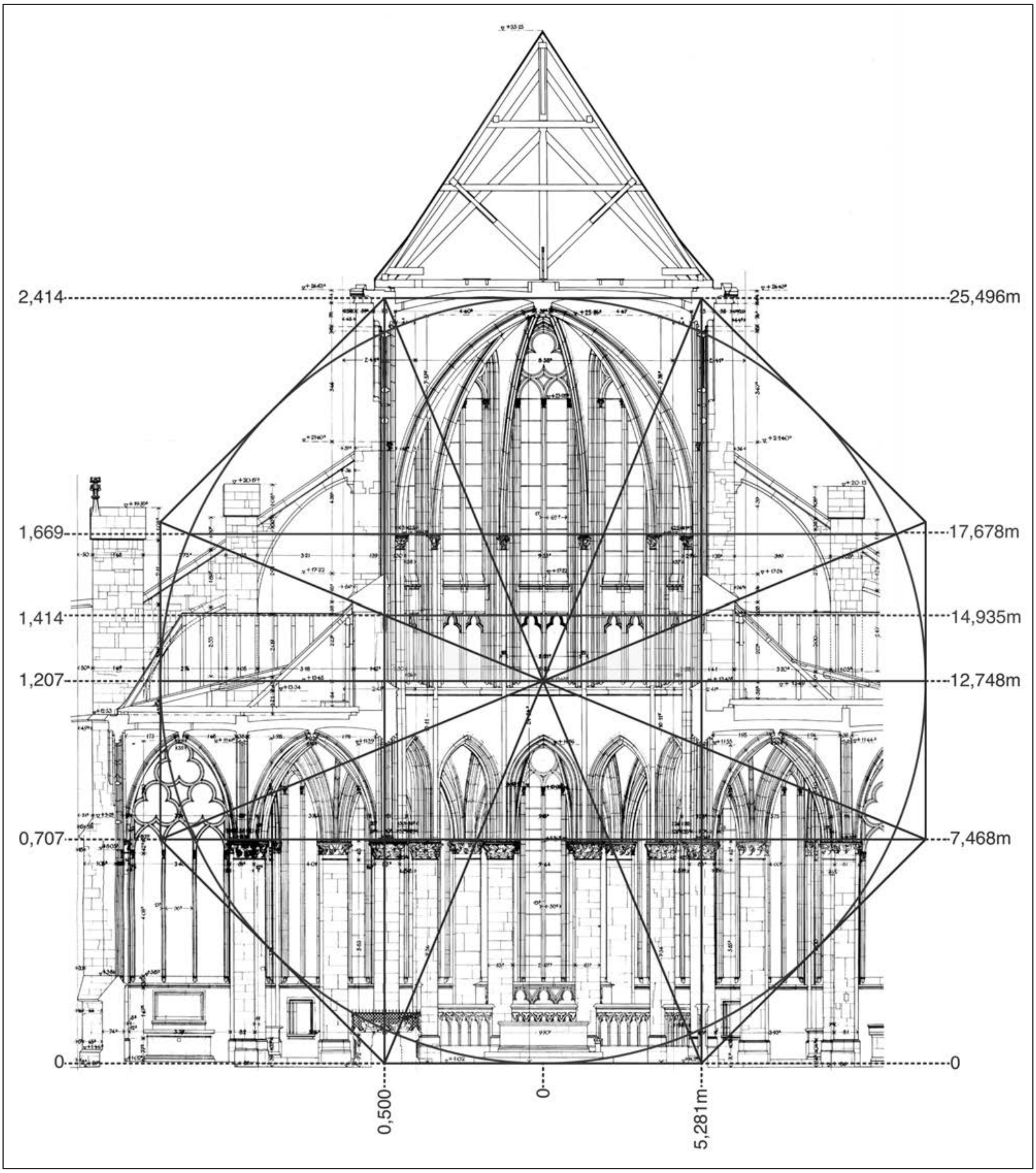

Fig. 10: Altenberg, choir section of Cistercian church, after Steinmetz (1911), as reproduced in Lepsky and Nussbaum, *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg*, vol. 1, with geometrical overlay by the author.

of the choir section are again set by a single great octagon, as **Figure 10** shows.

As at Reims and Clermont, the midpoint of the octagon aligns with the base of the triforium, instead of with the midpoint of the triforium, as it does at Prague. This variation helps to illustrate the flexibility that Gothic designers enjoyed, even when working within a crisply defined geometrical framework like that of the octagon. At Altenberg, as at Reims, the geometry of the elevation involved not just the main octagon, but also the circles related to it. So, while the tops of the arcade capitals coincide with the lower corners of the octagon, at a height equal to 0.707 of the main vessel span, the smaller capitals of the high vault fall at height 1.669, coinciding with the level were the rays to the octagon corners cut the circle inscribed within it. Many other crucial heights in the Altenberg section can be found by logical extension of this system, but the preceding examples should already suffice to demonstrate the relevance of the basic octagonal framework.

The case of Altenberg has the potential to reveal a great deal about Gothic building practice, because recent studies of the building are starting to show how members of the

Altenberg workshop used modular dimensions together with dynamic geometry to develop the design. This can be seen in both plan and elevation. At Altenberg, as at Cologne Cathedral, the overall groundplan of the chevet was set by a dodecagon.[40] In each case, one facet of this twelve-sided figure would correspond to a single radiating chapel. In Cologne, the geometry is particularly clear, with the array of chapels thus corresponding to exactly 7/12 of a regular dodecagon. In Altenberg, though, the geometry is less regular, because of a complex interaction between geometrical and arithmetical design modes. The relative widths of the choir, aisles, and buttresses were definitely set by the geometry of a regular dodecagon; the relationships are quite precise. But, while all of the columns of the chevet sit on the circles defined by these radii, their positions on these circles were not set by a regular dodecagon. Instead, as Norbert Nussbaum and Sabine Lepsky have demonstrated, they are separated by intervals of 2.5 column diameters, where each column diameter of 83 centimeters in turn equals 2.5 feet of 33.2 centimeters. These same units seem to have been used throughout the construction of the choir. In elevation, for example, the height to the top of the main arcade capitals can be expressed as 9 column diameters, or 9 x 0.830 m = 7.470 m; this almost perfectly matches the geometrically determined height seen with the octagon corner in **Figure 10**, which is $1/\sqrt{2}$ x the choir span, or 0.707 x 10.56 m = 7.468 m. Interestingly, too, the same geometrically determined heights seen in **Figure 10** continue to govern the elevation of the west façade at Altenberg, which was built in the fourteenth century using a slightly larger foot unit of 33.55 centimeters.[41] The use of these two distinct modular systems to approximate geometrically determined dimensions provides an excellent example of Toker's principle of 'pseudo-modularity'.

Conclusion

The preceding case studies illustrate several important points about the use of geometrical proportioning strategies in Gothic architecture. They show, first of all, that centuries of sophisticated tradition informed the work of late Gothic authors like Roriczer and Lechler, even if their writings were not eloquent enough to compete with the work of their Renaissance rivals. They demonstrate, moreover, that Konrad Hecht was wrong to dismiss the importance of geometry in Gothic form generation. Numerical and module-based thinking certainly played a role in Gothic design practice, but not to the exclusion of dynamic geometry. Instead, these were complementary strategies: sometimes geometrical constructions could be unfolded within modularly defined armatures, as in the Ulm ground plans; in other cases, modules could be combined to approximate geometrically determined proportions, as in the Altenberg choir elevation. Most fundamentally, though, these examples begin to hint at the rich variety of geometrical planning strategies employed by Gothic designers, which deserve far more detailed and rigorous exploration than they have received to date. With the increasingly widespread availability of reliable building surveys and CAD systems, and with the rapid progress of research on Gothic drawings, there is good reason to be optimistic that more of this kind of scrutiny will soon be forthcoming. Enough good work has already been done in this field, though, to demonstrate that Gothic architecture embodied a complex procedurally based formal order whose conventions governed the dynamic unfolding of geometry, rather than fixed canons of proportion like those seen in classical architecture. In this sense, Gothic designers anticipated the work of their twenty-first century successors, who are now beginning to use computer algorithms to explore similarly dynamic approaches to form generation. Research on Gothic geometry thus has the potential to enrich not only the scholarly discourse on medieval architecture, but also a larger and broader conversation about the character of architectural order and proportion.

Notes

1 'There are works of another sort that are called German, which differ greatly in ornament and proportion from the antique and the modern. Today they are not employed by distinguished architects but are avoided by them as monstrous and barbarous, since they ignore every familiar idea of order; which one can rather call confusion and disorder, for in their buildings, of which there are so many that they have contaminated the whole world, they made portals adorned with thin columns twisted in corkscrew fashion (vine tendrils), which do not have the strength to support a burden, however light. And so, above all their facades and their other decorative parts, they built one cursed tabernacle on top of the other, with so many pyramids and points and leaves that they do not stand, as it appears, not to mention being able to hold themselves up, and they have more the quantity of seeming to have been made of paper, than of stone or marble. And in these works, they made so many projections, openings, little consoles, and twining vines, that they threw the works that they built out of proportion; and often they reached such a height, by placing one thing on top of another, that the end of a door touched its roof. This manner was invented by the Goths, who, after the destruction of the ancient buildings and the dying out of architects because of the wars, afterwards built—those who survived—edifices in this manner'. Vasari, *Vite,* quoted in Frankl (1960: 290–291).
The original Italian reads as follows: 'Ècci un'ultra specie di lavori che se chiamano tedeschi, I quali sono di ornamenti e di proporzione molto differenti dagli antichi e dai moderni. Nè oggi s'ussano per gli eccellenti, ma son fuggiti da loro come mostruosi e barbari, dimenticando ogni lor cosa di ordine; che più tosto confusion o disordine si può chiamare, avendo fatto nelle lor fabbriche, che son tanto che hanno ammorbato il mondo le porte ornate di colonne sottili ed attorte a uso di vite, le quali non possono aver forza a reggere il p ìeso di che leggerezza si sia. E così, per tutte le facce ed altri loro ornamenti, facevano una maledizione de tabernacolini l'un sopra l'altro, con tante piramidi e punte e foglie, che, non ch'elle possano stare, pare impossibilie ch' elle se possano reg-

gere; ed hanno piu il modo da parer fatte de carta, che di pietro o di marmi. Ed in queste opera facevano tanti risalti, rotture, mensoline e viticci, che sproporzionavano quelle opera che facevano; e spesso con mettere cosa sopra cosa, andavano in tanta altezza, che la fine d'una porta toccava loro il tetto. Questo maniere fu trovatat dai Goti che, per aver ruinate le fabbriche antiche e morti gli architetti per le guerre, fecero dopo colo che rimasero le fabbriche di questa maniere'. See Vasari (1550/1878: 137).

2 This point is made at greater length, with analysis of many other architectural drawings, in Bork (2011b).

3 The subdivided columns designed by Michael Hansmeyer provide one recent example of such work. See 'Projects' on Hansmeyer's website, www.michael-hansmeyer.com. Fractals more generally have generated an immense literature, to which a seminal contribution was Mandelbrot (1977).

4 It is not even clear, in fact, that Villard was an architectural professional, although he evidently enjoyed access to the workshop of Reims Cathedral, of which he drew not only whole elevations, but also minute details such as pier and mullion sections. See, most recently, Barnes (2009).

5 In the language of systems theory, therefore, one can say that Gothic and classical architectural conventions embody the principles of 'process description' and 'state description', respectively. See Simon (1962). In a related vein, Gothic architects could be described as 'designing-in-time', to extend the model of 'building-in-time' described by Marvin Trachtenberg, while classical architects generally sought to construct embodiments of timeless order. See Trachtenberg's essay in this volume, and Trachtenberg (2010).

6 This development was likely catalyzed not just by the invention of the printing press, but by the publication of Italian architectural treatises, as discussed below.

7 'zuerleuteren [...] den anefang des auszgeczogens stainwerches wie vnd jn welcher mass das ausz dem grunde der geometrey mit austailung des zirckels herfurkomen'.

8 The Lechler illustrations are carefully discussed in Müller (1990: 90–94). For a useful discussion of the relationship between geometrical and modular design processes at Salisbury Cathedral, see Kidson (1993: esp. 62–75).

9 Unlike square rotation and quadrature, analogous relationships based on other polygons have received scant attention to date. The proportions of many octagonally symmetrical towers and apses, however, were clearly set by the relationship between octagons and their circumscribing circles. While a circle circumscribed around a square by quadrature has a diameter 1.414 times as great as the square's side length, the 'octature' operation gives a circle with diameter 1.082 times the octagon's width. Relations based on the circumscribing of circles around dodecagons, meanwhile, govern the proportions of the Cologne Cathedral apse. See Bork (2011b: 26, 98).

10 In mathematical terms, ϕ satisfies the equation $\phi=1/(\phi-1)$, and it has the value $(1+\sqrt{5})/2 = 1.618...$ Its importance for Gothic design has been effectively demonstrated by authors including Stephen Murray, who sees it as a crucial generator for the plan geometry of Amiens Cathedral, and Peter Kidson, who documents its use at Salisbury. See Murray and Addiss (1990) and Kidson (1993).

11 For provocative discussions of this rhetorical asymmetry and its consequences, see Crossley (1992) and Kavaler (2007).

12 For a concise and surprisingly compelling discussion of these contrasts between medieval geometry and Renaissance modularity in painting, see Bouleau (1963: 49–113). There was not, of course, a strict black-and-white division between these two design modes. For a case study of the overlap in architecture, see Cohen (2008). For a valuable perspective on the Renaissance as a purification of historicizing trends already evident in Italian medieval architecture, see Trachtenberg (1992).

13 Hecht's *Maß und Zahl in der gotischen Baukunst* first appeared as three successive issues of *Abhandlungen der Braunschweigischen Wissenschaftlichen Gesellschaft*: 21 (1969), 22 (1970), and 23 (1970). The complete study has been republished as a single volume by Georg Olms Verlag (Hildesheim, 1979). The more widely available book version includes the following passages cited here: the general critique of earlier literature, mostly on pp. 2–60; the critique of geometrical literature on the Freiburg tower in particular, (60–92); an appeal to Italian sources (130–171); Villard de Honnecourt (201–217); a modular approach to the Freiburg tower (334–361); Gothic drawings in general (381–387); the Ulm elevation drawings in particular (387–468).

14 Other ideological forces more complex than simple skepticism may well have informed Hecht's distrust of geometrical explanations for Gothic design. Since the geometrical sophistication of German Gothic design was a source of nationalist pride for authors such as Otto Kletzl who enjoyed favored positions in the Third Reich, this intellectual legacy likely appeared tainted after the Second World War. Hecht surely would have felt this particularly strongly, since he worked at the University of Braunschweig, where a strict and reductive modernism dominated the architecture school in the decades after the war, providing a strong critique of the Reich and its bombastic historicism. On Kletzl's career in the war years, see Labuda (2003). On the architecture school in Braunschweig, see Böttcher et al. (1995).

15 On Regensburg, see Hubel and Schuller (2010). For Cohen's work, see his essay in this volume and Cohen (2008).

16 See the articles in Bork, Clark, and McGehee (2011), especially Davis (2011). See also Neagley (1992) and Neagley and Davis (2000).

17 Major recent publications on Gothic drawings include the three imposing catalogs produced by Johann Josef Böker: Böker (2005) and Böker et al. (2011 and 2013). For a complementary geometrical perspective, see Bork (2011b). For medieval drawings more generally, see Holcombe (2009).

[18] The absolute scale of drawings, admittedly, can be affected by shrinkage or stretching of the parchment or paper on which they are drawn. In most cases, however, these effects appear to have been quite small. So long as the effects are uniform, moreover, the geometrical structure of the design remains unchanged.

[19] In some of the taller and narrower drawings composed of multiple parchment sheets, for example, the vertical axes required straightening, but such corrections do not affect the proportions of the individual sheets. For drawings, the proportions could be checked against first-hand measurements made in the relevant archives. For images of buildings, the proportions were checked against published survey data. For Freiburg and Ulm, for example, this data can be found in Hecht (1979).

[20] This quality of the computer models, unfortunately, does not translate onto the printed page, where all the lines in both the original drawing and the overlaid figures must appear as ink bands of finite width.

[21] The larger study is Bork (2011b).

[22] In some instances, in fact, the draftsmen appear to have used protective screens to keep their drawings from being punctured at key points where a compass had to be used repeatedly. In the drawing known as Rahn Plan B, which is preserved in Fribourg, Switzerland, a series of concentric arcs was carefully drawn, quite obviously with a compass, to describe the inner arch profiles of a flying buttress. There is, however, no hole or prick point at their geometrical center. This effect could have been achieved by temporarily attaching a small parchment patch atop the main drawing to shield the center point during the arc construction process.

[23] See, for instance, Friedrich (1962) and Koepf (1977). The first drawing holds inventory number 3549 in the Victoria and Albert Museum, while the second is preserved in Ulm's Archiv des Münsterbauamtes.

[24] On these two drawings in particular, see Böker et al. (2011: 38–40, 53–56). In the overall scheme proposed by the Böker group, Ulrich von Ensingen's contributions to the Ulm tower project are eclipsed to some extent by those of his successors, and by the work of his predecessor Heinrich Parler the Younger, to whom the group attributes a spectacular spire drawing now preserved in Regensburg. See Böker et al. (2011: 31–37). Significantly, however, the horizontal proportions of the drawing do not match the wide-aisled format of Ulm Minster; instead, they align perfectly with those of Regensburg Cathedral's thirteenth-century choir, as shown in Bork (2011b: 314). The dynamics of artistic exchange between the two workshops remain to be clarified, but it is clear that Ulrich von Ensingen established the overall format of the actual Ulm tower base, whose geometrical logic becomes readily comprehensible in the plan drawings preserved in London and Ulm.

[25] Coenen (1990: 95–96).

[26] The units shown in plain text along the left margin of the drawing are the same as those seen in **figures 2c** and **2d**; in other words, one such unit equals the span between the building centerline and the buttress axis measured at ground level. The italicized units along the right side of the drawing are 0.979 times as large, corresponding to the span between the pinched buttress axes higher in the tower, the locations of which were established in **Figure 2d**.

[27] The subtle inward pinching of the buttress axes can be seen lower in the drawing, at level 2.023, where small circles highlight the points of disjunction.

[28] The question of whether the tower was designed by one or two masters has long been disputed. For a geometrically based reading that supports the attribution of the upper and lower tower sections to two different masters, see Bork (2011b: 126–165; [forthcoming 1]). At the 2010 conference *Der Freiburger Münsterturm und sein europäischer Kontext*, Hans Böker and Anne-Christine Brehm argued that the tower as a whole was conceived together with its openwork spire by the thirteenth-century architect Erwin von Steinbach. For an account of the contrasting views presented at the conference, see http://www.badische-zeitung.de/kultur-sonstige/der-hochgelobte--35962036.html. Böker's revival of the one-master argument had already been presented less formally in publications such as www.kit.edu/mediathek/print_looKIT/Mit_KIT-Bauhistorikern_in_mittelalterlichen_Kirchen.pdf. In their most recent publications, however, Böker and his team suggest that the main period of spire planning at Freiburg came only after the completion of the tower base. See Böker et al. (2013: 70–105, esp. 80, 94–100). This position seems to mark a tacit willingness to accept a two-master chronology, although this is not stated as clearly as it might be.

[29] The emphasis here is on the dating of the design scheme shown in drawing 16.869, rather than on the dating of the drawing itself. The distinction is important, since many scholars see 16.869 as a fourteenth-century copy of a thirteenth-century prototype. See Bork (2011b: 143–146), Böker (2005: 165–166), and Böker et al. (2013: 89–93).

[30] Using Hecht's own measurements for the flange and buttress spans, the ratio of 15.71m to 12.39m is 1.2679. The triangular construction described here gives 1.2679, for accuracy to four decimal places. Hecht's postulated flange span of 50'6' and buttress span of 40 feet, by contrast, give a ratio of 50.5/40=1.2625

[31] On the origins of the openwork spire type, see Bork (2003). That article emphasizes the importance of the drawing known as Rahn Plan B, which presents a slightly elaborated variant of drawing 16.869, but the original of 16.869 was likely produced even earlier. All of the early drawings of the Freiburg spire and its variants, significantly, include features such as crocketed gables and compound pinnacles that relate very closely to those seen in the upper choir of Cologne Cathedral. This strongly suggests that the openwork spire idea was first developed with input from the Cologne workshop.

[32] This can be seen, in particular, at the height labeled 1.407 in **Figure 3**, where the large generating circle of the tower base cuts the buttress axes.

[33] See Bork (2011b: 144–146; [forthcoming 1]). This identification of the 16.869 scheme as the first surviving design for the Freiburg spire has also recently been accepted by Böker, who had formerly seen the drawing as an elaborated reinterpretation of the already completed structure. See Böker (2005: 165–66) and Böker et al. (2013: 89–93).

[34] For a more complete discussion of these steps, see Bork (2011b: 152–157).

[35] Here, as in Ulm Riss C and the Freiburg-like drawing 16.869, the spire is three times as high as it is wide. This simple arrangement contrasts with the numerical scheme proposed in Hecht (1979: 359). While Hecht was correct to note that the height of the spire pyramid at Freiburg does not relate to the full height of the structure by a perfect Golden Section ratio, the match is close enough to make one suspect that the designers of the tower superstructure may have had this relationship in mind, as proposed in Wangart (1972). The details of the tower design, however, were evidently determined by the more precise scheme illustrated here, and in Bork (2011b: 157–159).

[36] Ackerman uses the term 'incommensurable' in lieu of 'irrational', but the meaning is the same.

[37] On the Clermont-Ferrand section and its relation to an unexecuted late Gothic design for the cathedral's façade, see Bork (2011b: 390–400). On the façade drawing itself, see Davis (1983).

[38] See Deneux (1948) and Villes (2009). If the original elevation of Reims was indeed meant to fill the octagon exactly, this might explain why the clerestory illustrated by Villard de Honnecourt is shorter than that of the present building. See Bork ([forthcoming 3]; [forthcoming 4]). These studies build on the work of Nancy Wu (notably 1996 and 2002b).

[39] On Trier, see Bork [forthcoming 2]. For the early history of the Altenberg project in general, see Lepsky and Nussbaum (2005). For the elevation geometry of the choir and façade, see Bork's contributions to Lepsky and Nussbaum (2012, esp. 75–88).

[40] On Cologne, see Bork (2011a: 97–100).

[41] On the Altenberg choir proportions, see Lepsky and Nussbaum (2005, esp. 42–62), and Nussbaum (2003). On the nave and façade, see Lepsky and Nussbaum (2012).

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