RESEARCH ARTICLE

Decoding the Pantheon Columns

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The goal of this study has been to reconstruct the design principles underlying the construction of the Pantheon's portico columns as well as to demonstrate how digital investigation methods and models can be used to improve our understanding of ancient architectural knowledge. Thanks to the data of the Bern Digital Pantheon Model, a synthesis of all the scanned surface points obtained during a digitization campaign of the Karman Center for Advanced Studies in the Humanities of the University of Bern in 2005, we have been able to determine empirically the column profiles of the portico with unprecedented precision. A second stage of our investigation involved explaining the profile of the column shafts by a construction model that takes into account the parameters recommended by Vitruvius and design methods such as those that can be found in the construction drawings discovered at Didyma. Our analysis shows that the design principles of the portico's columns can be successfully reconstructed, and has led to the surprising result that at least two different variants of a simple circle construction were used. Finally, we have been able to deduce from the distribution of the different profile types among the columns that the final profiles were designed and executed in Rome.

Introduction

The shafts of the Pantheon's portico columns are composed of granite monoliths, forty Roman feet tall, which were excavated from two Egyptian quarries. The eight light gray columns of the front row of the portico originated from the imperial quarry at Mons Claudianus (Stevens 1924), while the pink column shafts of the middle and back rows (four in each row) probably came from the Assuan region (De Fine Licht 1968: 40). The sixteen colossal shafts, each weighing about fifty metric tons, were brought by ship from Egypt to Rome. Although they were by no means the largest monoliths used in Roman architecture (indeed, there are indications that the portico of the Pantheon was originally intended to be furnished with even larger columns; see Wilson Jones 2000: 204–211; Wilson Jones 2009), their transportation and erection would nevertheless have posed a serious challenge to the architects of the time.

Although the architectural and historical significance of the columns of the Pantheon's portico are undisputed, the question of how they were designed remains a moot point. Particular attention has been given to the design problem of the so-called 'entasis', the slight vertical curvature of the column shaft. Gorham P. Stevens was the first to explore systematically this architectural feature in Roman architecture (Stevens 1924). To determine the design principles of Roman columns, he measured and geometrically analyzed columns from a variety of well-known Roman buildings, coming to the conclusion that Roman column design could reach a considerable degree of complexity. For example, Stevens believed that the Pantheon's portico columns exhibit a profile that is based on two tangent hyperbolas. More recently, the design and execution of the entasis has been reassessed by Mark Wilson Jones, who seriously doubts Stevens' results (Wilson Jones 1999). Nonetheless, his counterproposals remain speculative in many respects, principally because in many instances he was not granted access to re-measure the examples analyzed by Stevens.

The current study, which is based on the data gathered for the *Bern Digital Pantheon Project* (Graßhoff et al. 2009), analyzes the geometry of the Pantheon's portico columns with the aim of developing a hypothesis on the geometrical design of the entasis. The model developed to explain the measured column profiles is based on the parameters that Vitruvius gives for the design of columns in his treatise, *De architectura libri decem* (*Ten Books on Architecture*), and on the scale ratios of the construction drawings discovered at the temple of Apollo in Didyma, Turkey. Furthermore, in this study we investigate the challenges of reconstructing the knowledge stocks of the explicit spatial, geometrical and practical knowledge that is needed to execute theoretical models. The alternatives to descriptive and practical forms of knowledge do not help us to ascertain what kind of knowledge and expertise was needed to produce the Pantheon's portico columns. Rather, as we work in the field of the epistemology of architecture, we realized that spatial knowledge requires many different forms of knowledge. Reconstructing such knowledge thus requires methodological tools that are capable of describing these forms of knowledge and providing appropriate means to justify the results.

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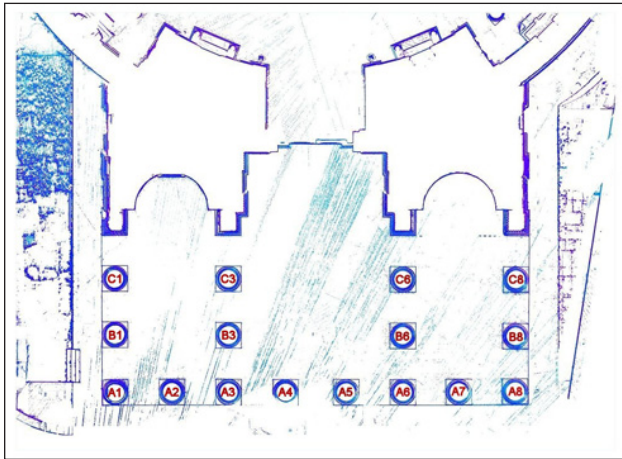 gerd.grasshoff@hu-berlin.de, berndt@kulturtechniker.de

Fig. 1: A slice of the 3D model of the Pantheon's portico at a height of 2 meters (BDPP0750). The columns are labeled by row name (row A = front row; B = middle row; C = back row) and are numbered from left to right (when viewed from the front of the portico).

We hope that this study demonstrates how digital research methods can be used to answer questions in the field of archaeology and architectural history. The reconstruction of different forms of knowledge stocks requires appropriate presentation and justification strategies. This requirement implies that the relevant data and the algorithms used must be provided in an accessible form, so that readers can reproduce and manipulate the analyzed data. Therefore, all the relevant justification materials have been published via the online facilities of the project and are openly accessible.[1]

Data acquisition and model construction

The basis for the analysis of the geometrical properties of the Pantheon's portico columns are 3D point sets from the digital model of the Bern Digital Pantheon Project. The model comprises all the inner and outer surfaces of the building and is composed of forty-three partial scans of different parts of the structure. The measurements were gathered during the course of two digitization campaigns carried out in 2005 and 2007 using a Leica HDS3000 3D long-range scanner.[2]

During the second stage of the project, the forty-three partial scans were integrated into a universal model using a common coordinate system. Afterwards, the point cloud was 'frozen', so that we have a reliable reference model for future research. The purpose of merging the partial scans into a unified model was not to produce visualizations of the building or virtual camera flights through the model. Rather, the objective was to enable us to compare and combine different parts of the building and examine the results of different investigations using a defined data set and a common coordinate system.

To analyze how the sixteen portico columns were made, the coordinates of each column were extracted from the reference model. The sixteen data sets comprise between one and two million measurement points. The combined files have a size of between twenty-nine and sixty-four megabytes and are stored in the universally readable

ASCII format, so that the data can be analyzed and visualized using any application. **Figure 2**, for example, shows a visualization of all sixteen data sets with an interactive notebook, created using *Mathematica*, a software program that offers the visualization and analysis of a wide range of numerical functions.

All the data sets are available to the public over the internet, so that anyone can test the original measurements of the presented results free of restrictions.[3] The Pantheon Project adheres to a strict open-access policy. Not only are the project's publications available on the internet, but any additional research materials that are needed to justify the results may be used and downloaded from the project's website.[4]

Vitruvius on column design

Although reservations about the value of Vitruvius's treatise, *De architectura*, as a source of archaeological and architectural topics have repeatedly been expressed (Knell 2008: 114; Wilson Jones 2000: 35ff), it is at least widely acknowledged that Vitruvius correctly identified the main factors relating to the design of buildings. The precise values of the dimensions and proportions he recorded may have been inconsistent even at the time of writing, but his explanations nevertheless provide invaluable insights into the way ancient architects worked and the strategies they could adopt to solve practical design tasks.

According to Vitruvius, column design is governed by two main factors. On one hand, the form of a column is determined by the distance between the individual columns, which is itself dictated by the appearance, strength and usability of the different types of temples, or *species aedium* (III 3.1). On the other hand, the shape of a column is set by the rules pertaining to the three ancient orders of classical architecture: the *doricum, ionicum*, or *corinthium genus* (IV praef.2). The basic principle is that '[t]he larger
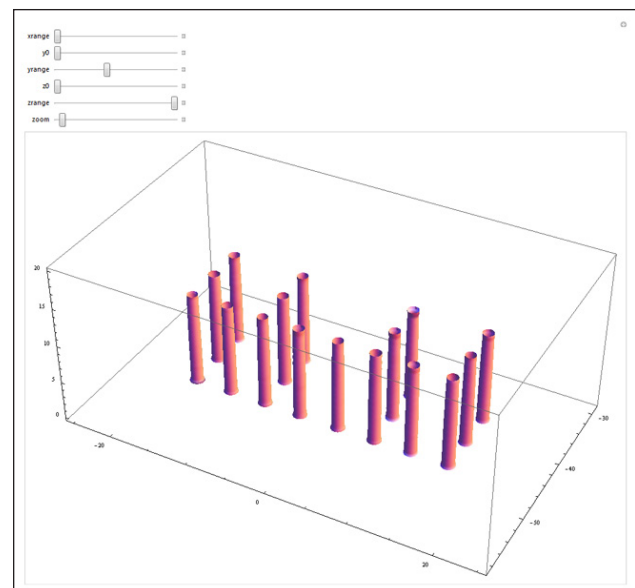

Fig. 2: A *Mathematica* notebook file with a spatial visualization of the coordinate sets of the portico columns (BDPP0749). The x- and y-axes both indicate distances measured in meters.

the space between the columns, the greater the diameters of the shafts must be' (III 3.11).[5]

According to this principle, the smaller the distance between the columns, the more slender the column proportions must be. Inappropriately slender or thick columns should, likewise, be avoided. One extreme type of temple is the *pycnostyle*, in which the space between each column is only the width of one-and-a-half columns, while the column has a length of ten column diameters. The other extreme is marked by the *araeostyle*, in which the intercolumniation is equal to more than three diameters and the length of the column is eight column diameters. In between is the *eustyle* temple, which, with an intercolumniation of two-and-a-quarter column diameters and a length of nine and a half modules, is, according to Vitruvius, 'the most laudable' type of temple (III 3.8, 10).

Irrespective of the particular order, the form of a column is also governed by its absolute height and its position in the architectural context:

> The corner columns […] must be made thicker than the others by one-fiftieth of their diameter, because they are cut into by air on all sides and therefore seem more slender to the viewer. Thus where the eye deceives us, reasoning must compensate. (III 3.11)

Similarly, the contraction of the shaft below the capital is independent of the particular order. The shorter the column, the slighter the tapering must be. Short columns (that is, columns less than fifteen Roman feet) must have an upper diameter in a 5:6 ratio with respect to the diameter of the lower part of the shaft. As for tall columns, Vitruvius notes:

> Those columns that are between forty and fifty feet high [i.e., the class of the columns of the Pantheon's portico] should likewise be divided into eight parts, and the top of the shaft, just below the capital, should be contracted into seven of these parts. (III 3.12)

Like the thickening of the corner columns, the different contraction factors for columns of different lengths are meant to compensate for optical illusions:

> For our vision always pursues beauty, and if we do not humor its pleasure by the proportioning of such additions to the modules in order to compensate for what the eye has missed, then a building presents the viewer an ungainly, graceless appearance. (III 3.13)

The tapering of the column does not follow a straight line but has the form of a slight curve:

> At the end of the present book I shall record the illustration and method for the addition made to the middles of columns, which is called *entasis*

[stretching] by the Greeks, and how to execute this refinement in a subtle and pleasing way. (III 3.13)

From the context one can infer that, like the other refinements, the inclusion of an entasis is meant to add to the beauty of a building. Unfortunately, neither Vitruvius' illustration (*forma*) referred to in the above quotation nor the description of the method (*ratio*) has survived.

Most of Vitruvius' recommendations regarding the length, contraction, and thickening of columns refer to the form of the column shaft (*scapus*). But while his rules for contraction and thickening may be applied to the shaft, we can only infer its length. The one explicit statement relating to the length of the shaft alone is the recommendation that shafts of Ionic columns should have a length of eight and a half modules (V 9.4.), although this value applies specifically to porticoes of civic buildings. The shafts of temple columns should be less slender, and therefore longer. But nowhere do we find any precise specifications. Rather, it seems that the length of the column shaft must be treated as a dependent value. From the description of the proportions of Ionic columns in the third and fourth books, we can deduce a shaft length of slightly more than eight (8 1/6) lower diameters, a value that also applies to temple columns of the Ionic and Corinthian orders.[6]

Analysis of the portico columns

To analyze the geometry of the portico columns, the coordinate sets were cut into thin slices parallel to the column axis. For every slice, the radius and center were calculated with a best-fit algorithm that approximates small circles to the column slices. Thus, for the first time, questions on the design and construction of ancient columns have been analyzed on a comprehensive empirical basis. While former approaches have relied on a comparatively small number of measurement points of representative column samples from different buildings (e.g. Stevens 1924; Wilson Jones 1999), the current study not only takes into account the complete data set of an entire portion of a building but also applies these data to the analysis of the column in its entirety.

The profiles can then be plotted and compared with an interactive model that was implemented using the *Mathematica.*[7] **Figure 3** shows a screen shot from the interactive comparison sheet with the profiles of columns A4 and A5, the two central columns of the first row of the portico (see **Figure 1**). For graphic purposes, the actual shaft profiles are shown horizontally, and start at a height of about one meter along the coordinate system along the x-axis. A very slight curvature can be discerned between the lower and upper ends of the shafts. The steep segments on the left and right sides represent the apophyge inferior (**Fig. 3**, left) and the apophyge superior (**Fig. 3**, right) of the column shafts, which are the curved joints between the column shaft and its base and capital, respectively.

The columns of the portico all have very similar profiles, with the exception of the three badly damaged columns on the eastern flank (A1, B1 and C1), which were
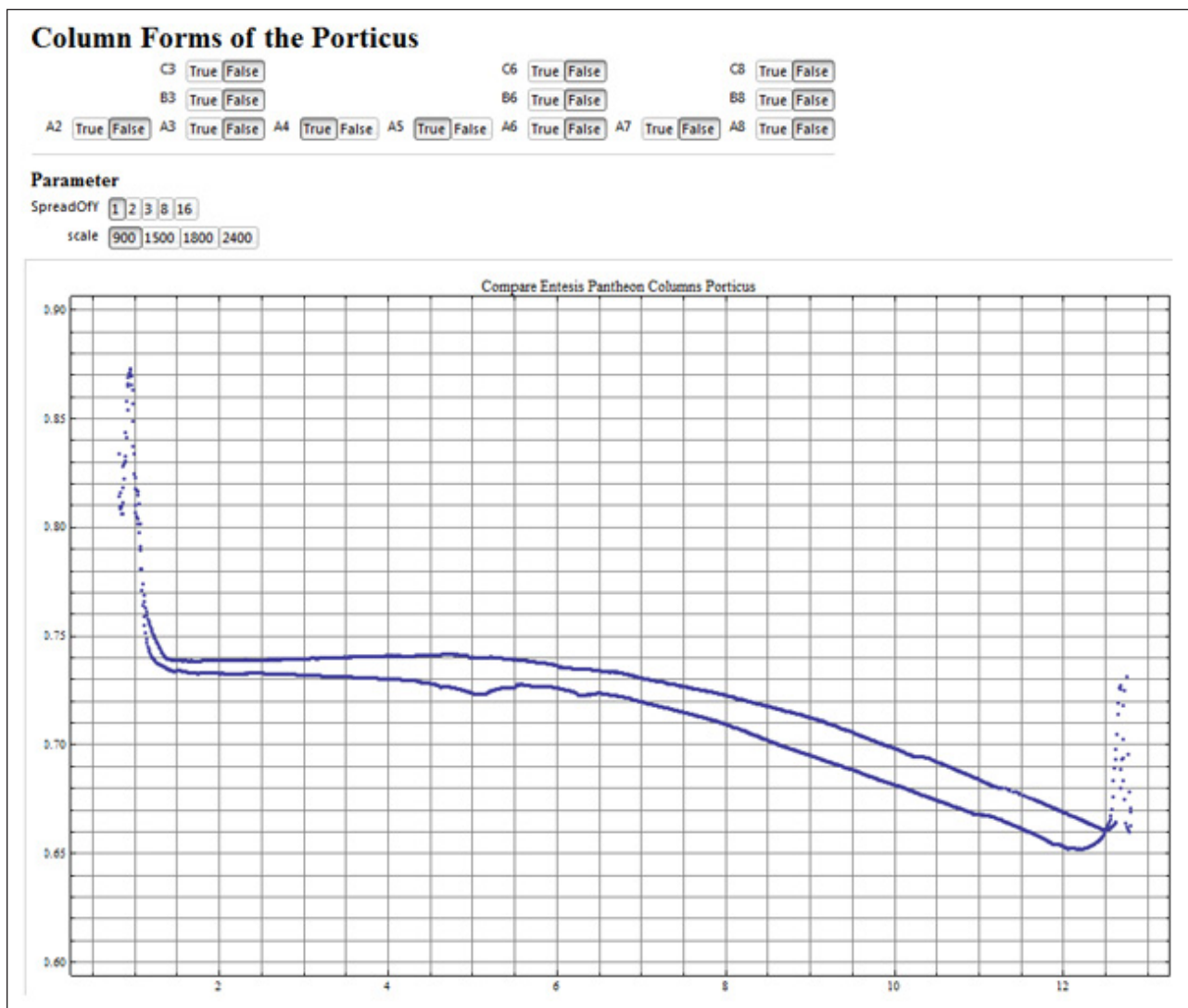
Fig. 3: A *Mathematica* notebook file for comparing the column radii (BDPP0748). The x- and y-axes both indicate distances measured in meters.

taken from the nearby Baths of Nero in the seventeenth century as part of restoration work carried out on the damaged portico (De Fine Licht 1968: 241–242). Since the repairs date from the modern period, these three columns have not been investigated any further in our study. In **Figure 4**, the profiles of the remaining thirteen original portico columns all lie within a width range of about 0.02 meters, which starts on the left with a lower shaft radius of between 0.72 and 0.74 meters, and ends on the right with an upper radius of between 0.63 and 0.65 meters.

The values of the main construction parameters of the Pantheon's portico columns are summarized in **Table 1**. The lower shaft radius refers to the straight elongation of the lower part of the profile onto the base of the shaft. In accordance with ancient design practice, the inferior apophyge has not been taken into account. Analogously, the location of the upper shaft radius was identified as being at the point where the upper part of the profile intersects with the upper end of the column shaft. This means that the empirical minimization can, but need not, coincide with the upper radius.

The average lower shaft radius amounts to 0.733 meters, while the average upper shaft radius comes to 0.642 meters. In calculating the average values, columns A8 and B3 were omitted as they are considerably thicker than the other columns. The thickness of column A8 can be explained by the fact that it is a corner column (see Vitruvius, III 3.11). However, the reason for the above-average thickness of column B3 remains unclear. The overview shows that the difference between the lower and upper radii is nearly always 0.09 meters in all the columns, which corresponds well to the value for the tapering of columns of between forty and fifty Roman feet mentioned by Vitruvius.[8] The thickening of the corner column A8 (approximately three per cent) also lies within the dimensions recommended by Vitruvius (III 3.11).

If one assumes that the shafts were not only proportioned according to Vitruvius' concept of *symmetria* — the idea that all the major dimensions of a building should be integer multiples or simple fractions of a common module and should also include integer values for the building height and column diameters — then the shaft dimensions are a good starting point for determining the length
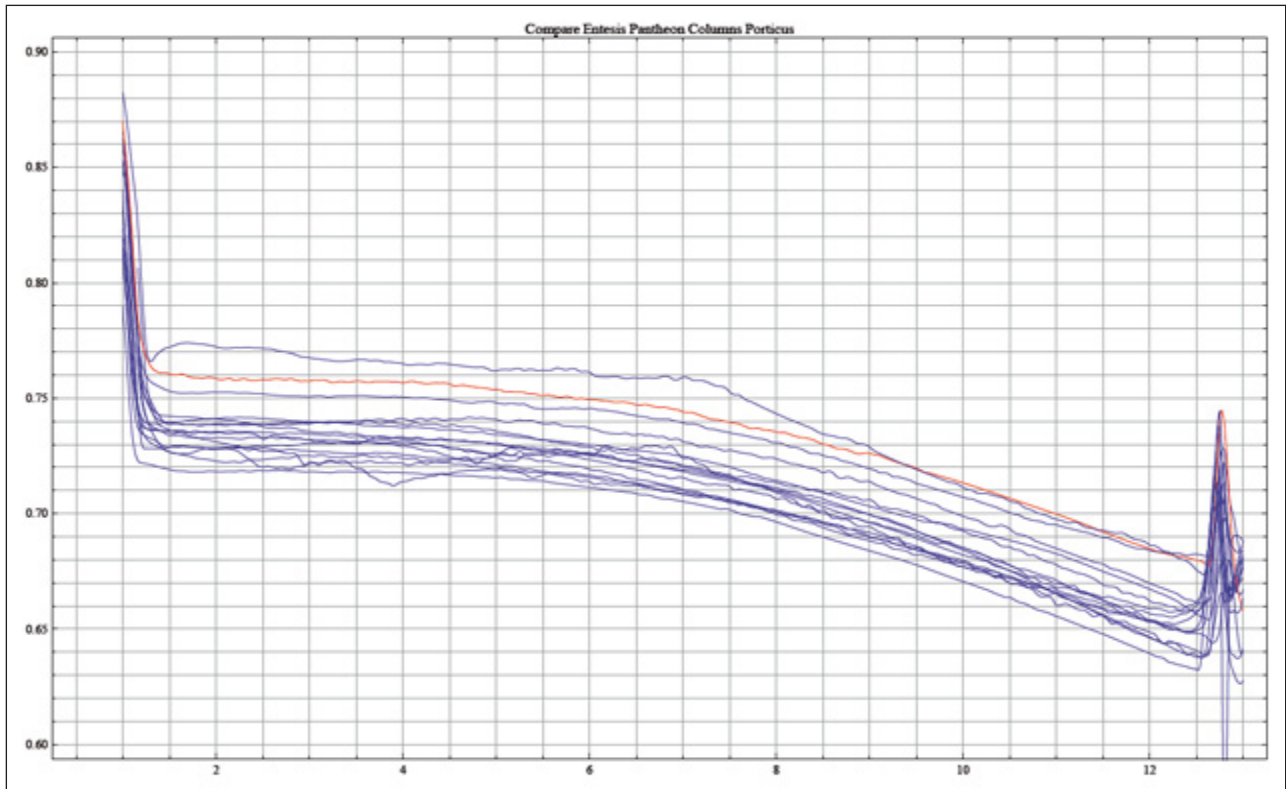
Fig. 4: A set of the curves of all the portico column radii as a function of their height, except those of columns A1, B1, and C1.

of the Roman foot used at the Pantheon. With a lower shaft diameter of five feet and an average (lower) column radius of 0.733 m ± 0.0072, we arrive at

$$1\text{ft} = 0.293 \text{ m} \pm 0.0029 \text{ m}.$$

A forty foot shaft is thus 11.7 meters tall.[9] The length of the Pantheon foot thus lies at the lower end of values previously suggested for Roman buildings (Wilson Jones 2000: 72). If other relevant dimensions (the diameter or height of the cupola, the width of the portico, and so on) were to be analyzed, the value could be rendered even more precisely.

Reconstruction of the design principles

The results of our analysis of the Pantheon's portico columns suggest that, with the exception of the corner columns, all the columns were designed according to the same principles. Nonetheless, the uniformity of the profiles and the tiny variations, in fractions of millimeters, remain surprising in light of the absolute sizes of the columns and their two different provenances (**Fig. 4**). The passages on entasis in *De architectura* suggest that there existed in antiquity well-known methods for designing slightly curved profiles. Numerous attempts were made to reconstruct these methods during the Renaissance, although none of them gained widespread acceptance.[10]

The entasis construction plan of Didyma

Our knowledge of ancient design strategies has improved dramatically since the discovery of the 'plan archive' at Didyma in 1979, which has greatly facilitated our analysis.[11] Among the drawings discovered by Lothar Haselberger on the foundation walls of the Apollo temple at Didyma are

Column	r_l	r_u	r_l − r_u
A2	0.738 m	0.648 m	0.090 m
A3	0.730 m	0.632 m	0.098 m
A4	0.733 m	0.640 m	0.093 m
A5	0.741 m	0.655 m	0.086 m
A6	0.741 m	0.654 m	0.087 m
A7	0.735 m	0.643 m	0.092 m
A8	(0.760) m	(0.672) m	(0.088) m
B3	(0.753) m	(0.663) m	(0.090) m
B6	0.725 m	0.642 m	0.083 m
B8	0.736 m	0.637 m	0.099 m
C3	0.729 m	0.637 m	0.088 m
C6	0.718 m	0.627 m	0.091 m
C8	0.739 m	0.649 m	0.090 m

Table 1: The main construction parameters of the Pantheon's portico columns (measurements in meters). r_l = lower shaft radius; r_u = upper shaft radius; r_l − r_u = difference between lower and upper shaft radii. Columns A8 and B3 were not taken into account when calculating the average values as they are considerably thicker than the other columns.

several drawings relating to the construction of the colossal columns (**Fig. 5**).

With regard to the problem of the design of the entasis, one drawing, located on the eastern end of the northern wall, is particularly important (**Fig. 6**).

Fig. 5: Photograph of the foundation walls of the Apollo temple at Didyma, containing the construction drawing interpreted in **Figure 6**.

This particular drawing is 1.2 meters high and about 1 meter wide, and, like all the other drawings, it was scratched very finely into the surface of the marble. The drawing shows the profile of half a column. Contrary to first impressions, it does not represent the lower part of a column but the geometrical construction of the whole column, excluding the capital. The lower part, complete with torus, rod, and apophyge inferior, is drawn full scale. The horizontally hatched area in the upper part of the drawing depicts the column shaft with intentionally distorted proportions. While in the horizontal direction the shaft is drawn full scale, in the vertical direction the compression ratio is 1:16 (Haselberger 1980: 200).

The vertical line (i) on the right side represents the axis of the column. On the left side another vertical line (f) runs parallel to the axis, at a distance equaling that of the lower shaft radius. The lower end of the shaft is marked by the last of the horizontal lines (d1), which is slightly longer than the rest of the hatching. The upper end of the shaft is defined by one of the horizontal lines at the top, which are all considerably longer than the lower parallel line (d1). The lower and upper end points of the profile are connected with an inclined line (h). Above this line is a faintly curved arc (g), which Haselberger has plausibly identified as being a circle segment. It runs through the upper end point of the profile and is tangent to the vertical line on the left (f) (Haselberger 1980: 199). The center of the circle segment has not been

identified, and thus the question of how the circle segment was executed remains unresolved.

The profile of a full-scale shaft is obtained by extracting the construction profile by a ratio of 1:16, which turns the circle segment into an elliptical segment. The horizontal lines on the shaft represent the column radius as a function of the column height. The construction drawing is used as a 1:1 scale guide for the shape of the column. Thus we can define the complex entasis as being the result of a drawn spatial operation. By using the full scale in the horizontal direction, the radius for a given height can be transferred directly from the drawing onto the work piece. Hence, the construction was not used to produce numerical data for the column form and cannot be interpreted as an analogue calculator.

Geometrical description

Figure 7 describes geometrically the entasis construction drawing found at Didyma, where r_l and r_u denote the lower and upper radii of the column shaft, a is the difference between the two, r_D indicates the radius of the construction circle, whose center lies on the same level as the lower end of the column shaft, and h is the height of the shaft in the Didyma drawing. In this construction
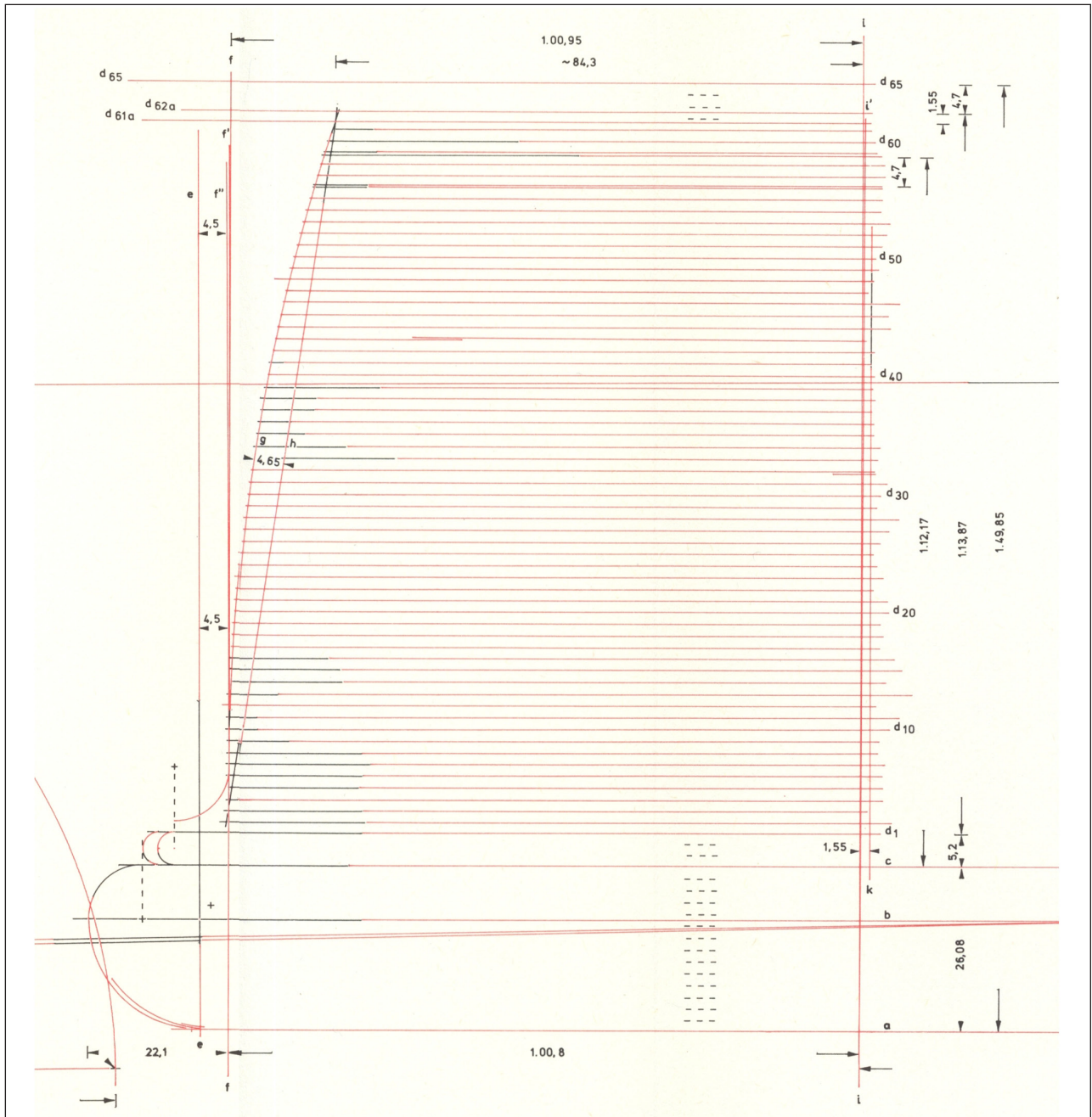
$$r^2_D = r_D - a^2h^2$$

Fig. 6: A copy of the column construction drawing at Didyma. In the horizontal direction, the drawing has been drawn full scale. In the vertical direction, the shaft has been compressed by the ratio 16:1. Red line: visible scratch line on the wall. Black line: conjectured continuation of an existing line. Dotted lines: additional lines by Haselberger (Haselberger 1980: Beilage 1, Abbildung 1; detail).

$$r^2_D = r^2_D - 2r_D a \, a^2 h^2.$$

We thus arrive at

$$2r_D a = a^2 h^2$$

from which it follows that

$$r_D = \frac{a^2 h^2}{2a}.$$

These calculations mean that the radius of the construction circle is solely determined by the height of the column shaft (in the Didyma drawing) and the difference between the lower and upper shaft radii. With a shaft length of eight lower diameters,

$$h_s = 8 \times 2r_l,$$

as found in the Pantheon's portico columns and deduced from Vitruvius, the length of the compressed column shaft amounts exactly to that of the lower radius, if one assumes that the compression ratio is 1:16:

$$h = r_l.$$

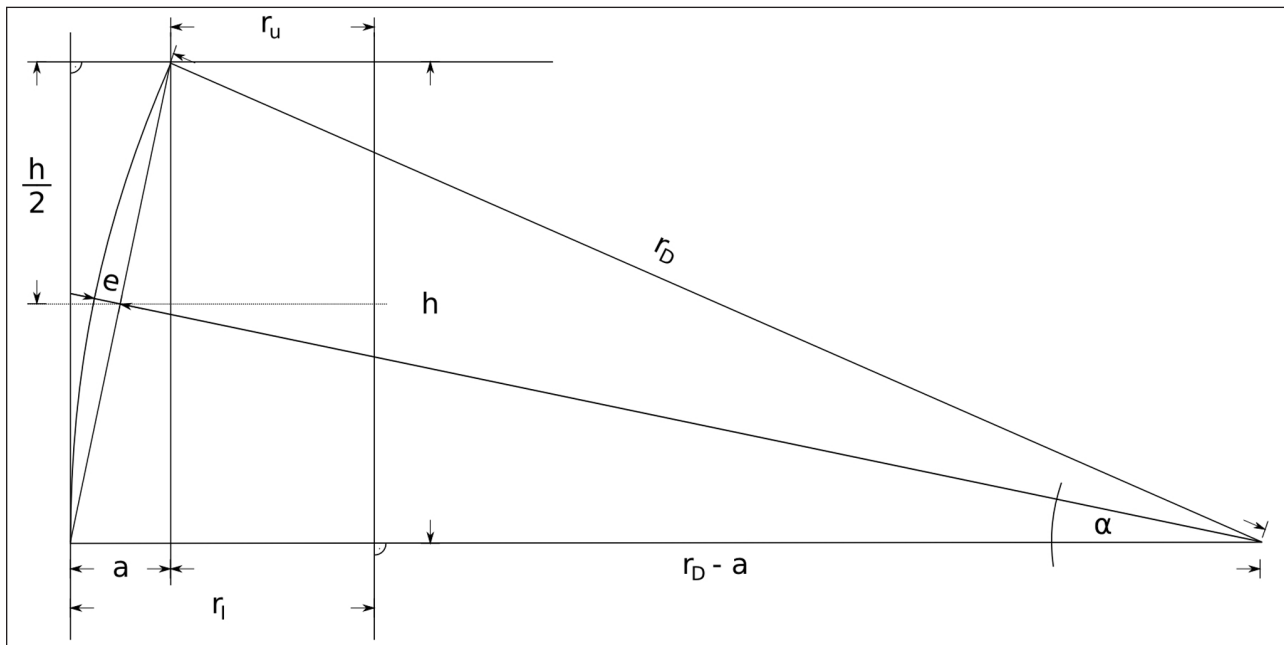With a tapering of the shaft of 1:8 of the lower diameter,

Fig. 7: A geometrical description of the Didyma construction drawing.

$$a = \frac{1}{8}r_l,$$

the radius of the construction circle comes to

$$r_D = 4\frac{1}{16}r_l.$$

This calculation means that the radius of the construction circle of the Pantheon's portico columns would be about four times the size of the diameter of the lower shaft, if it had been designed according to the method used at Didyma.

Construction model
It seems plausible that the curved profile of the Didyma construction drawing is a circle segment whose center lies on the baseline of the shaft, but there are no clues as to how this circle was constructed. In principle, one can envisage at least two distinct approaches: a construction with a given radius (see above) and various geometrical constructions without a predefined radius.

The simplest approach using a given radius would have been to measure the distance on the baseline. However, the baseline of the Didyma construction drawing is not long enough. Another way to determine the center of the construction circle would be to draw circles around the upper and lower end points of the profile. Probably the easiest way to do so would have been to fix a piece of string twice the length of the construction radius at the lower and upper end points of the profile. To determine the center of the construction circle, one would only have to tighten the string.

If the radius of the construction circle were not known in advance, one could draw the perpendicular bisector on the straight-line segment connecting the lower and upper end points of the profile and determine where it intersects

with the shaft's baseline. Such an approach might explain the straight line between the lower and upper end points of the profile. But in the reproduction of the drawing published by Haselberger, there is no evidence that there was a perpendicular bisector on this straight-line segment (h in **Figure 7**). Moreover, the baseline is too short for this kind of construction.

As there is no conclusive evidence of how the circle segment was constructed, neither in Vitruvius nor in the Didyma drawing, no assumptions on its construction have been made in the model developed to analyze the profiles. The model's only premise is that the profile of the Pantheon's portico columns was based on a circle segment and on the assumption that this segment was stretched by a ratio of 1:16. The model has been implemented as a dynamic *Mathematica* notebook, which allows the user to overlay interactively the empirical shaft profiles with different circle segments.[12] These circles are defined by at least three support points for which a best-fitting circle was calculated (**Fig. 8**). The support points had to be chosen in such a way that they corresponded to specific points of the profile's curve.[13]

Column A7, for example, has — except for damages sustained to the lower part — a very smoothly curved profile (**Fig. 9**, left). However, on closer examination, the profile reveals straight-line segments, which enables us to make inferences about the production process. If one defines the curved points between the straight-line segments as support points and calculates a circle with a least mean squares algorithm, the resulting circle segment corresponds to the chosen points as well as to the empirical profile.

By contrast, the profile of column C8 consists of three straight segments joined at comparatively sharp angles (**Fig. 9**, right). If one again locates the measuring points at the respective curved points and calculates the corresponding best-fit circle, one finds that the profile of column C8 can also be deduced from a circle-segment construction.
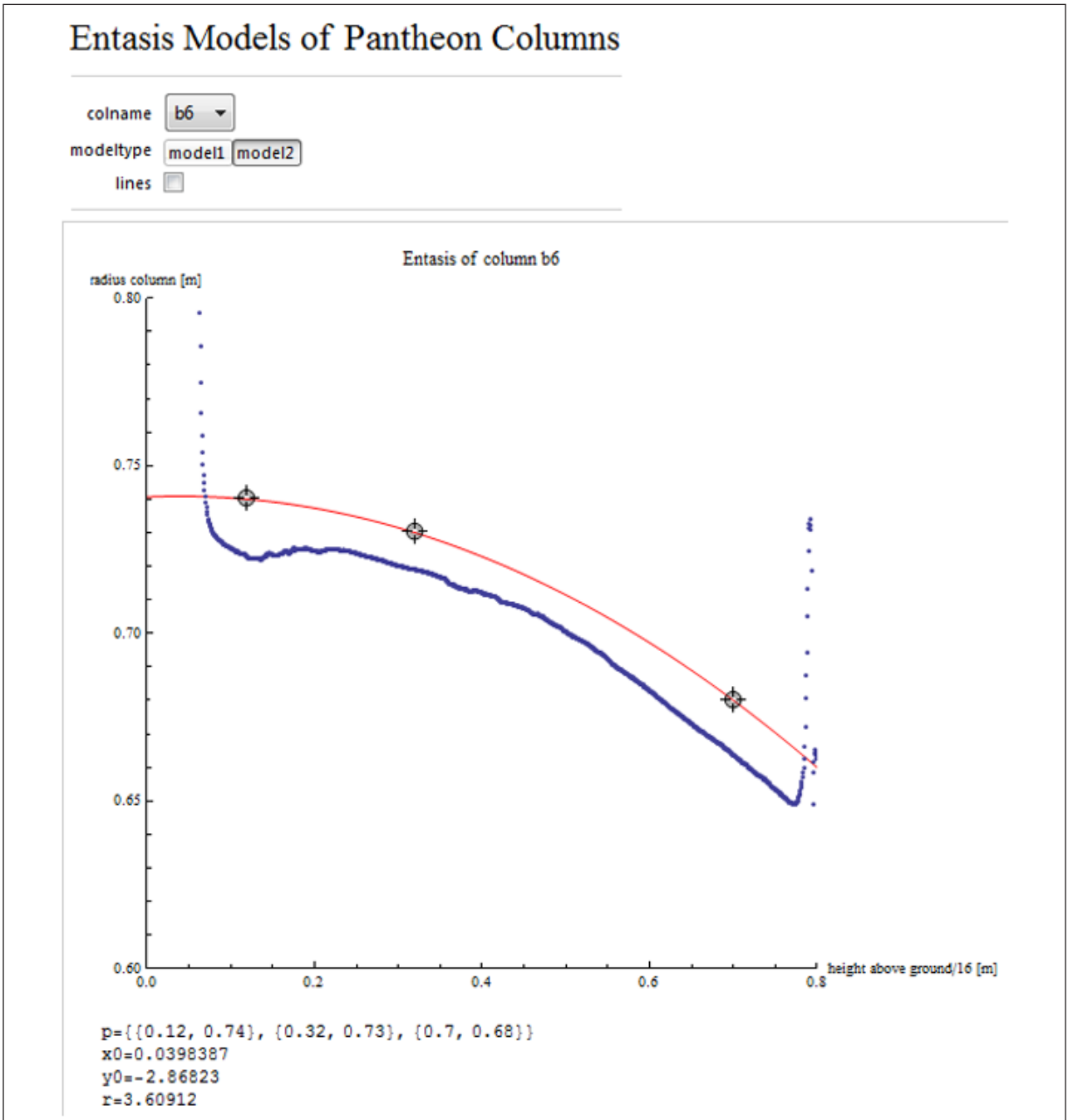
Entasis Models of Pantheon Columns

colname [b6 ▼]

modeltype [model1] [model2]

lines ☐

Entasis of column b6

radius column [m]

p={{0.12, 0.74}, {0.32, 0.73}, {0.7, 0.68}}
x0=0.0398387
y0=-2.86823
r=3.60912

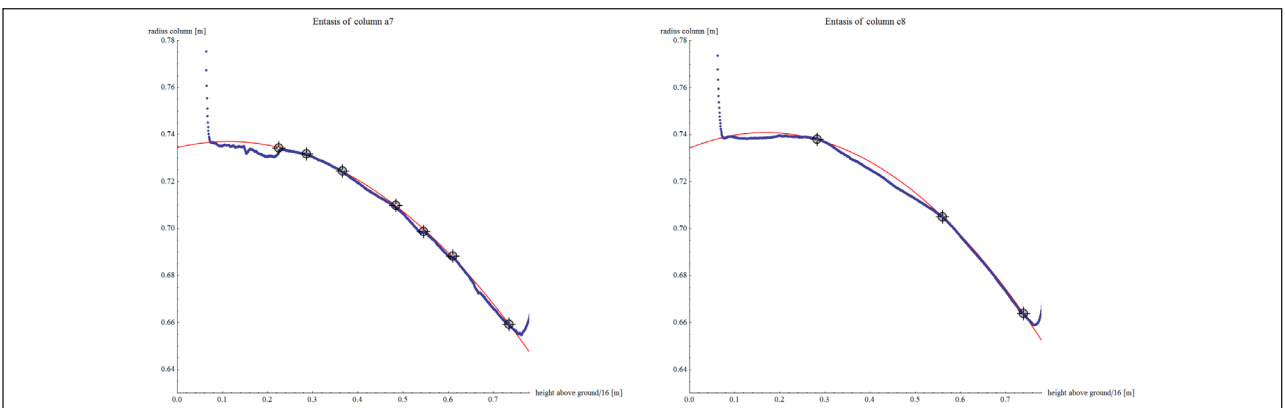**Fig. 8:** An interactive model for circle approximations (BDPP0752).

Fig. 9: Profiles of columns A7 (left) and C8 (right) compressed by a ratio of 1:16 with an overlaid circle segment. The circle segment was calculated using a best-fit algorithm on the basis of the indicated support points.
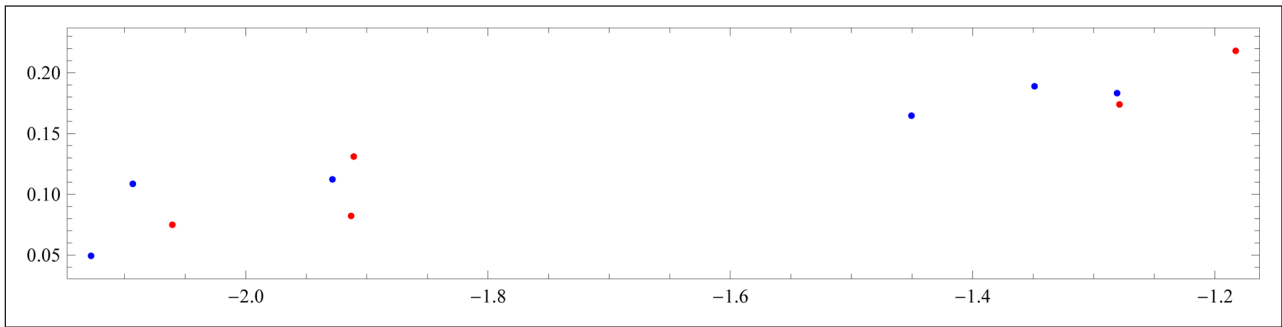
Fig. 10: A distribution of the circle centers with the coordinates (y0, x0). Blue: columns of the first row. Red: columns of rows B and C. Columns A1, B1, and C1 were not examined as they are restorations of the modern period.
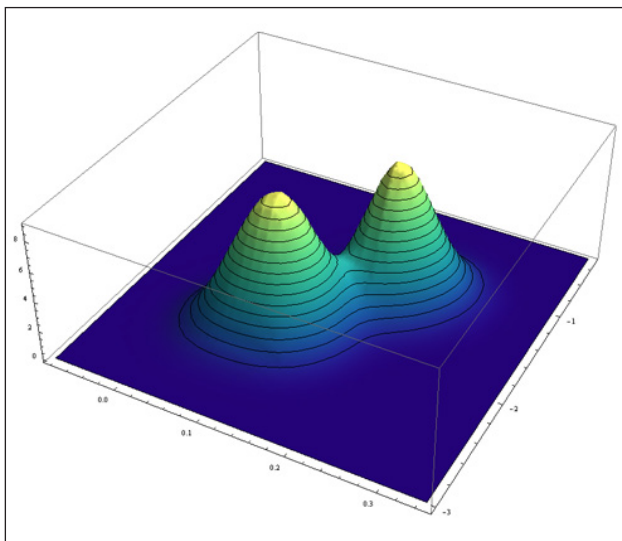

Fig. 11: A probability distribution of the circle centers.

However, the individual segments have been interpolated in a much coarser way than in column A7. Therefore, the actual profile shows a greater deviation from the geometrical construction, especially in the center.

A systematic comparison of all the profiles of the original columns reveals that all of them may have been derived from a circle construction. That there are in fact 'fitting' circle segments for *all* the profiles furthermore confirms the assumption that the construction of the Pantheon's portico columns was based on a ratio compression of 1:16, just as at Didyma.

The distribution of the construction circle centers is shown in **Figures 10** and **11**. The x-axis shows the distance of the circle's center from the column axis, while on the y-axis the distance from the baseline is shown. The results were entirely unexpected: Contrary to our original assumptions and to current opinion (Stevens 1924; Wilson Jones 1999), there is no common design principle for the whole set of portico columns under consideration. Rather, the distribution clearly reveals that at least two distinct construction methods for the Pantheon's portico columns were used.

One group is defined by circle centers with an offset of 1.9 to 2.1 meters from the shaft axis, which corresponds to a radius of 2.6 to 2.8 meters (**Fig. 11**). The vertical distance of the center from the baseline amounts to 0.05 to

0.13 meters. On the basis of these values, one can deduce that this group represents a baseline construction similar to that found at Didyma (see **Figures 6** and **7**). The radii of the construction circles are about four times the size of the lower shaft radius (4 x 0.73 m = 2.92 m). The deviation of the empirical data from the model can be explained by an unintended displacement of the center points, which is not improbable if one takes into account the acute-angled character of the construction.

The other group of circle centers is defined by a distance of 1.2 to 1.45 m from the shaft axis. Their offset from the baseline amounts to 0.16 to 0.22 meters (see **Figure 11**). Because of the distance of the centers from the axis and their distance from the baseline, it is unlikely that this group of columns rests upon a baseline construction. Rather, the positioning of the center points was probably the result of a conscious design decision.

The most obvious characteristic of this group of columns (A4, A5, B3, C6, and C8) is a nearly straight vertical profile segment in the lower part, which becomes a curved line only considerably above the base of the shaft. It is worth noting that this curved segment is not a tangent to the lower straight part but is based on a circle segment that runs through the lower and upper end points of the profile. As shown in **Figure 12**, the lower part of the circle segment, which protrudes over the vertical line segment, is 'cut off' (dashed arc) and replaced by a straight line. The center of the circle segment is located at half the height of the lower straight-line segment.

Figure 13 shows the distribution of the circle centers with a common scale for the x- and y-axes and the straight line connecting the lower and upper end points of the profile, together with its perpendicular bisector. *All* the center points lie, with only a small scattering, on the perpendicular bisector. The diagram thus clearly demonstrates that all the shaft segments are, in fact, based on a circle-segment construction, even those that actually started off with a straight-line segment.

The systematic analysis of the shaft profiles has led to another surprising result. As noted previously, the columns of the portico's first row, and those of the middle and back rows, originate from different Egyptian quarries (see above, Introduction). The remains of monolithic shafts preserved in ancient quarries suggest that, to a large extent, they were profiled while still in the quarry
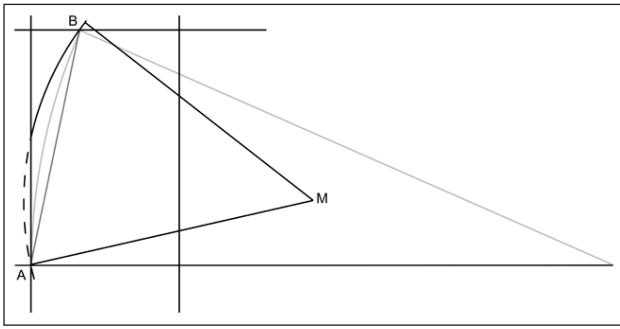
Fig. 12: A schematic representation of the second type of construction. As in the baseline variant (gray), the construction circle runs through the lower and upper end points of the profile (A, B). But as its radius is smaller, its center point (M) has moved upwards in relation to the baseline. The lower segment of the profile is 'cut off' where it extends beyond the vertical line, which runs parallel to the shaft axis for a distance equaling the radius of the lower shaft.
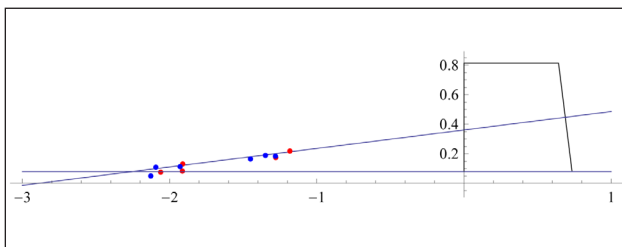

Fig. 13: A distribution of the circle centers with the x- and y-axes sharing a common scale. The blue line is the perpendicular bisector on the baseline, which connects the lower and upper end points of the shaft profile.

(Wilson Jones 1999: 246). Therefore, it would be logical to assume that the two different types of profiles might be linked to the two distinct provenances. However, the distribution of the column profiles shows that both profile types can be found in the columns coming from both provenances. We can conclude, therefore, that the final profile was only applied once the shafts had reached their final destination, where at least two distinct variants of a circle-segment construction were used.

Methodologically, it is worth emphasizing where our explanation differs from previous attempts. Compared with the aforementioned analysis of Roman entasis by Gorham P. Stevens (1924), the current approach has *not* been concerned with finding the best-fitting curves for empirical profiles. Rather, our aim has been to find a causal explanation for the form of the Pantheon's portico columns. On one hand, such an explanation must comprise a geometrical construction drawing that includes clearly defined parameters that are relevant for the resulting shaft profile. Furthermore, the method must be acceptable historically and realizable using simple tools in an ancient building yard. Therefore, it is vital that the explanation elucidates how — based on this construction — the column shafts could have been produced in practice.

The circle-segment construction similar to the Didyma drawing is a method that can be executed using simply

a compass and ruler. Its main construction parameters are the lower shaft radius, the well-defined integer ratio between the diameter and length of the shaft (1:16), and the difference between the lower and upper shaft radii, which is characteristic for columns of a certain size. Vitruvius observed and confirmed the values and relevance of all these parameters in *De architectura*. A model of the production process, which takes into account deviations of the empirical shaft form from the construction profile, is developed in the following section. It must be emphasized here, however, that the quality of the explanation does not depend on the conformity between the geometrical curve and the empirical profile but rather on the reasons given to explain their deviations.

Practical realization of the shaft profile

The relevant phases of the construction process can be reconstructed from the monolithic columns that were left unfinished in the quarries. After the raw shaft had been cut at the quarry, a cylinder was carved out of the cuboid. Then the profile of the entasis was transferred to the work piece by carving circular trenches into the column at specific, well-controlled distances. Finally, the protruding material was removed and the surface smoothed (**Fig. 14**).

It seems that the monolithic shafts were roughly worked on while still in the quarry because a tapered shaft could be more than ten per cent lighter than an unworked cylinder. With a mass of between fifty and fifty-five metric tons for shafts to fit the Pantheon's portico columns, this substantial reduction in size would have considerably facilitated shipping. Nevertheless, too fine an execution would have increased the risk of damage during transportation (Wilson Jones 1999: 247). So, as previously mentioned, it can be assumed that the final, finished profile was not applied until after the shafts had arrived at their final destination. The evidence of the Pantheon's portico columns, where two distinct construction variants can be found that do not correspond to the different provenances, confirms this assumption.

Characteristic errors

During the different phases of the construction process a number of errors seem to have occurred from which we can draw conclusions about how ancient columns were made. Errors might have occurred when the construction diagram was drawn, when the relevant dimensions were transferred onto the work piece, or during the manual execution of the profile. The different types of error are characteristic of their causes.

The most characteristic error of the Pantheon's portico columns is probably the varying thicknesses of the shafts. All the ancient columns have approximately the same profiles: The lower and upper end points of the shaft profiles are connected by a slight curve, while the difference between the upper and lower radii remains constant. However, the thicknesses of the shafts differ by up to 0.06 meters (see **Figure 4** and **Table 1**).[14] A comparable variation of the radius cannot be found within the profile of a single column. Thus, one can infer that the profile design

Fig. 14: An unfinished monolithic column shaft from Ain Dougga in Tunesia, with clearly visible trenches, which were carved into the column shaft as part of its profile.

was carried out independently of the absolute width of the column. At the same time, a comparison of the profiles reveals that the final form of the shaft was dependent upon the positioning of just a single point. If this point were misplaced, all the dependent points would be misplaced, too. Such an error might occur, for example, if the support points of the profile had been positioned using a molding tool or a piece of string that had been fastened either too near or too far from the work piece. In such a case, all the support points would lie too near or too far from the column axis, and consequently the resulting shaft would be made too slender or too thick.

The polygon interpolation of the profile represents a second type of error. When transferred from the construction drawing onto the work piece, marking the various radii, the circle segment became an elliptical segment by being stretched by a ratio of 1:16. Depending on how many support points were used to define the profile, the curve became more or less smooth in shape. The positioning of the support points can be discerned from characteristic curves in the profile. The more support points there are, the smoother the resulting profile will be (**Fig. 4**). The entasis of column B8, for example, was executed extremely diligently and matches the reconstructed profile design in almost every respect. When fewer support points were used, the curves between the straight segments become more obvious, as in the profile of column C8, for example, which seems to have only two support points (**Fig. 9**). Consequently, the points of the curve are much more pronounced and the deviations of the empirical profile from the construction drawing are much more distinct.

On some of the columns we observed a third, barely discernible, type of error in the profiles. The otherwise diligently executed profile of column B8, for example, is, at its center, slightly more slender than intended, a variation that is most probably the result of imprecisely positioned support points. Similar variations within a range of a couple of millimeters can also be found in some of the other columns. They may either have been caused by the design

being imprecisely transmitted onto the work piece or by errors that occurred during the manual execution of the profile. However, these errors lie within an order of magnitude that is barely visible to the naked eye and that can only be properly observed with high-resolution measuring techniques.

Summary

The results of our analysis of the Pantheon's portico columns may be summarized as follows. The shafts have a ratio of 1:8. From their average height of 11.7 meters and their average lower shaft radius of 0.733 m ± 0.0072 m, a Roman foot length of 0.293 m ± 0.0029 m can be determined. The shafts taper continuously to the top for about 0.09 meters, which corresponds to a contraction factor of 0.876 relative to the average lower radius.

The integer proportions of the shafts, 1:8, clearly confirm that the *symmetria* concept observed by Vitruvius was applied. The continuous tapering of the shafts furthermore confirms the relevance of the lower and upper shaft diameters for the column design. The empirical tapering factor of 0.876 matches extremely precisely the value given by Vitruvius for columns of the size of the Pantheon's portico columns ($7/8 = 0.875$). The thickening of the corner column A8 of about three per cent is slightly greater than the value given by Vitruvius, yet is still in the same order of magnitude. All in all, the design principles described by Vitruvius are confirmed by our measurements of the Pantheon's portico columns.

We have been able to establish that a circle-segment construction, similar to that found at Didyma, is the design principle of the columns' entasis. As in the Didyma drawing, the portico columns were probably designed with a full-scale radius and a vertical ratio contraction of 1:16. The entasis profile can be characterized as a circle segment which turns into an elliptical segment when stretched by a ratio of 1:16.

Two types of construction principles can be distinguished from our examination of the portico columns. One is characterized by construction circles with a center lying on the baseline, as at Didyma. In this case, the radius of the construction circle comes to about four lower shaft radii and the profile is smoothly curved along the shaft's full length. The second type of profile starts with a nearly straight vertical segment that becomes a more pronouncedly curved circle segment only considerably above the baseline. The radius of the construction circle is considerably smaller than in the first type, and the center point of the construction is shifted vertically upwards. Nevertheless, in this type of construction the profile is also defined by a circle segment that runs through the lower and upper end points of the profile.

Moreover, the systematic analysis of a self-contained set of similar columns in the Pantheon portico shows that even columns of similar type may vary much more than was previously expected. The typical differences found between the columns allow one to arrive at detailed conclusions regarding the design and manufacturing process. From the evidence of two typical variants of the design method, one may conclude that the final

profile was not applied until after the shafts had arrived in Rome, where at least two different workshops with two different design methods were responsible for preparing the columns.

Finally, it should be emphasized that our findings could not have been achieved without digitization technology. The results can only be fully justified and understood by accessing the high-resolution measurement data and the resulting models. Therefore, the high-resolution data sets, plots, and interactive models provided on the project's website are not only didactic materials but are also an integral and constitutive part of our research results.

Acknowledgements

Biographies

Gerd Graßhoff
Formerly Professor of History and Philosophy of Science at the University of Bern in Switzerland, and now Professor of History of Ancient Science at the Humboldt University of Berlin, Gerd Graßhoff's research fields cover the history of ancient science from Babylonian astronomy to modern times, methods of scientific discovery and philosophical models of causal reasoning.

Christian Berndt
After completing studies in the history of art, comparative literature and history of science at the Humboldt University of Berlin and the University of Bern, Switzerland, Christian Berndt was awarded a PhD from the University of Bern for his work on the epistemological implications of digital research methods and digital publishing. He has been a member of the Digital Pantheon Project since March 2008, and is currently working on the relationship between architecture and science in antiquity.

Notes

[1] Go to www.digitalpantheon.ch.

[2] For the project history, data acquisition and development of the model, see Albers (2009).

[3] From www.digitalpantheon.ch/BDPP0716 to www.digitalpantheon.ch/BDPP0731.

[4] Go to www.digitalpantheon.ch.

[5] All the quotations from Vitruvius have been taken from the edition Howe and Rowland (1999). In addition, the German translation by Fensterbusch (1981) and the French edition and translation by Gros (1999) have been used as reference material.

[6] For the height of the base, see Book III 5.1; for the height of the capital, see Books III 5.5 and IV 1.1; for the total length of the column, see Book IV 1.8.

[7] Available online at http://www.digitalpantheon.ch/BDPP0748. For the sake of performance, the respective data have already been embedded in the notebook

player files published online, so there is no need to open a connection to the reference model or to load the point sets. The zero level of the coordinate system corresponds approximately to the level of the portico floor but is arbitrarily determined by the definition of the coordinate system. The height of the column above the zero level of the coordinate system is shown on the x-axis, while the calculated radius of the column for a given height is displayed on the y-axis. In order to compare the different column profiles, the profiles can be individually displayed and hidden by selecting the relevant boxes in the upper part of the notebook file.

[8] III 3.12: The upper diameter of columns of forty to fifty feet in height should amount to $\frac{7}{8}$ (0.875) of the lower diameter. The factor used empirically for the Pantheon's portico columns ($r\_u : r\_l$) comes to 0.876.

[9] $40 \times (0.293 \pm 0.0029)$ m $= 11.72 \pm 0.116$ m.

[10] For reconstruction attempts in the early modern era, see Becchi 2008.

[11] First published in Haselberger (1980), (1983). For the latest discussions, cf. Senseney (2011).

[12] Available online at http://www.digitalpantheon.ch/BDPP0752.

[13] Unlike the Didyma drawing, the plots within the model (see **fig. 8**) have been rotated to the left by 90°. The profiles of the different columns can be selected using the menu in the upper left of the notebook file (*colname*). One can choose between a model with a predefined radius (select *model1* box) and a circle with a variable radius (select *model2* box). The lower and upper shaft radii as defined by Vitruvius can be displayed by selecting the checkbox labeled '*lines*'. The slider *shift* allows one to move the empirical profile in a vertical direction in order to compensate for varying shaft radii. The circle segment (red) may be manipulated interactively by clicking and dragging the different support points. Additional support points can be added by clicking the left mouse key while holding down the Alt key. Support points can be removed by holding down the Alt key while clicking onto an already existing point.

[14] See *Bern Digital Pantheon Project* notebook file BDPP0748 at www. digitalpantheon.ch/ BDPP0748.

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